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About A LEVEL PHYSICS

Paper 4

(TOPICAL)

About **Learning Corner**

This section reveals the extra/relevant information that are not required as part of the model answers but to expand the student's knowledge in the respective fields. At the same time, it induces eagerness and interest into their learning process.

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CONTENTS

☞ A LEVEL

Topic 2

Gravitational Fields

2.1 Gravitational field

Candidates should be able to:

1 understand that a gravitational field is an example of a field of force and define gravitational field as force per unit mass

2 represent a gravitational field by means of field lines

2.2 Gravitational force between point masses

Candidates should be able to:

1 understand that, for a point outside a uniform sphere, the mass of the sphere may be considered to be a point mass at its centre

2 recall and use Newton's law of gravitation $F = \frac{Gm_1m_2}{r^2}$ for the force between two point masses

3 analyse circular orbits in gravitational fields by relating the gravitational force to the centripetal acceleration it causes

4 understand that a satellite in a geostationary orbit remains at the same point above the Earth's surface, with an orbital period of 24 hours, orbiting from west to east, directly above the Equator

2.3 Gravitational field of a point mass

Candidates should be able to:

1 derive, from Newton's law of gravitation and the definition of gravitational field, the equation $g = \frac{GM}{r^2}$ for the gravitational field strength due to a point mass

2 recall and use $g = \frac{GM}{r^2}$

3 understand why g is approximately constant for small changes in height near the Earth's surface

2.4 Gravitational potential

Candidates should be able to:

1 define gravitational potential at a point as the work done per unit mass in bringing a small test mass from infinity to the point

2 use $\phi = -\frac{GM}{r}$ for the gravitational potential in the field due to a point mass

3 understand how the concept of gravitational potential leads to the gravitational potential energy of two point masses and use $E_p = -\frac{GMm}{r}$

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TOPIC 2 *opening ....*

Question 1

(a) Define *gravitational potential* at a point. [2]

(b) The Earth may be considered to be an isolated sphere of radius R with its mass concentrated at its centre.
The variation of the gravitational potential ϕ with distance x from the centre of the Earth is shown in Fig. 1.1.

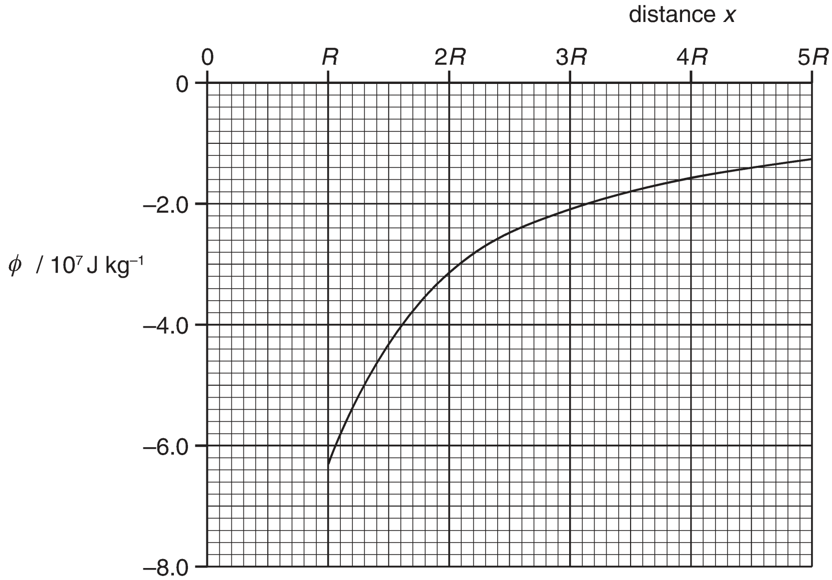

Fig. 1.1

The radius R of the Earth is 6.4×10^6 m.

(i) By considering the gravitational potential at the Earth's surface, determine a value for the mass of the Earth. [3]

(ii) A meteorite is at rest at infinity. The meteorite travels from infinity towards the Earth.
Calculate the speed of the meteorite when it is at a distance of $2R$ above the Earth's surface. Explain your working. [4]

(iii) In practice, the Earth is not an isolated sphere because it is orbited by the Moon, as illustrated in Fig. 1.2.

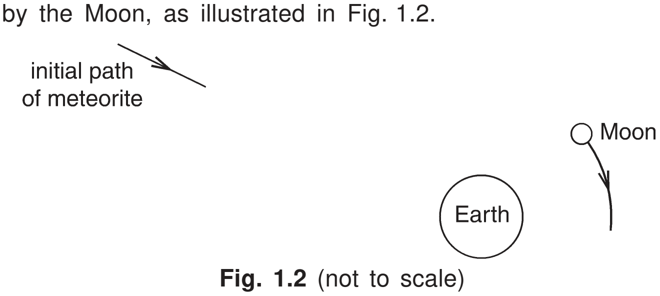

Fig. 1.2 (not to scale)

The initial path of the meteorite is also shown. Suggest two changes to the motion of the meteorite caused by the Moon. [2]

[J10/P4/Q1]

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Suggested Solution:

(a) Amount of work done to bring a unit mass from infinity upto a point in the gravitational field.

(b) (i) From graph at R, $\phi = -6.3\times10^{7}\ \mathrm{J\,Kg^{-1}}$

$$\phi = -\frac{GM}{R}$$

$$-6.3\times10^{7} = \frac{(-6.67\times10^{-11})(M)}{6.4\times10^{6}}$$

$$M = 6.0\times10^{24}\ \mathrm{kg}$$

(ii) By Principle of conservation of energy ,
loss in potential energy = Gain in kinetic energy

$$(\Delta\phi)m = \frac{1}{2}mv^{2}$$

$$v^{2} = 2\frac{GM}{3R}$$

$$v^{2} = 2(2.1\times10^{7})$$

$$v = \sqrt{4.2\times10^{7}}$$

$$v = 6.48\times10^{3}\ \mathrm{ms^{-1}}$$

(b) (ii)

$$v^{2} = \frac{2(6.67\times10^{-11})(6.0\times10^{24})}{3(6.4\times10^{6})}$$

$$v = 6.46\times10^{6}\ \mathrm{ms^{-1}}$$

(iii) 1. The velocity of the meteorite will increase.

2. The meteorite deviates from its straight line path.

Question 2

(a) Define *gravitational field strength*. [1]

(b) An isolated star has radius R. The mass of the star may be considered to be a point mass at the centre of the star.
The gravitational field strength at the surface of the star is g_S.
On Fig. 1.1, sketch a graph to show the variation of the gravitational field strength of the star with distance from its centre. You should consider distances in the range R to $4R$.

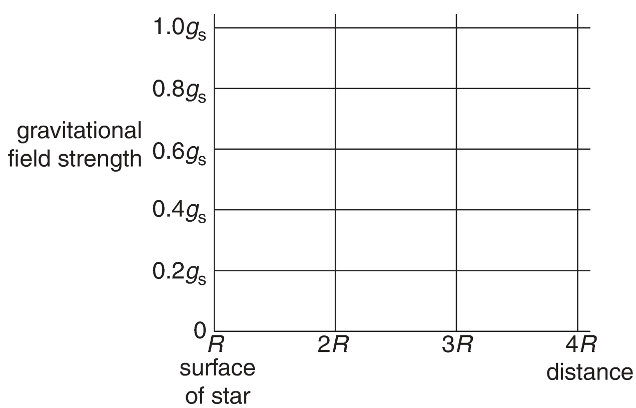

Fig. 1.1 [2]

(c) The Earth and the Moon may be considered to be spheres that are isolated in space with their masses concentrated at their centres.
The masses of the Earth and the Moon are 6.00×10^{24} kg and 7.40×10^{22} kg respectively.

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The radius of the Earth is R_E and the separation of the centres of the Earth and the Moon is 60 R_E, as illustrated in Fig. 1.2.

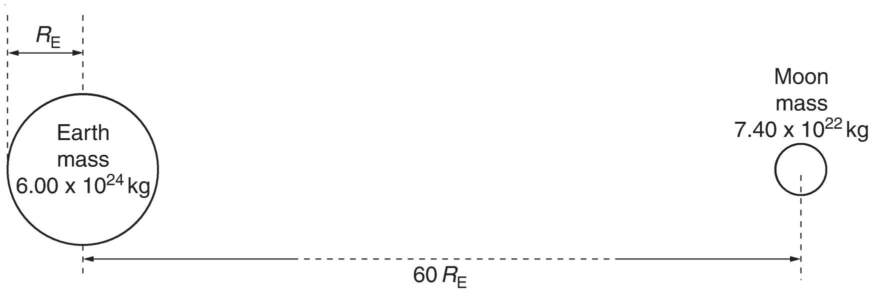


Fig. 1.2 (not to scale)

(i) Explain why there is a point between the Earth and the Moon at which the gravitational field strength is zero. [2]

(ii) Determine the distance, in terms of R_E, from the centre of the Earth at which the gravitational field strength is zero. [3]

(iii) On the axes of Fig. 1.3, sketch a graph to show the variation of the gravitational field strength with position between the surface of the Earth and the surface of the Moon.

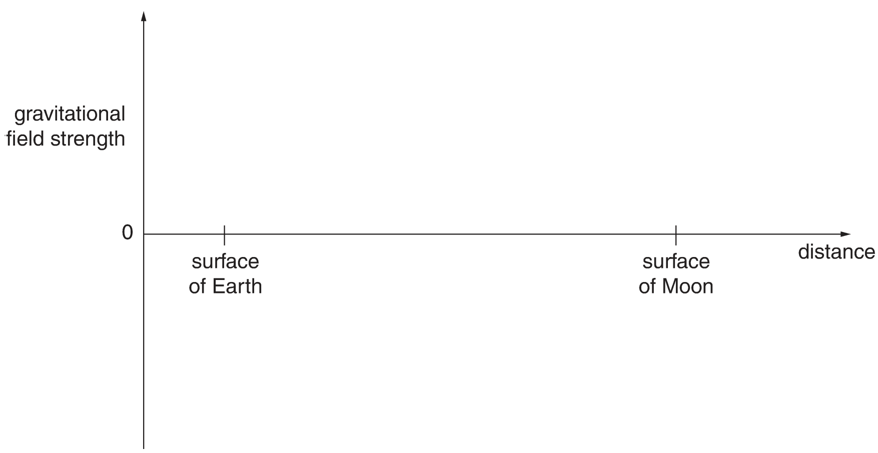


Fig. 1.3

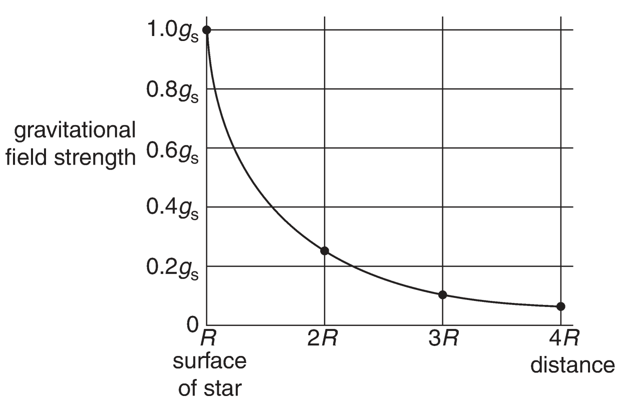
[3]

[N10/P4/Q1]

Suggested Solution:

(a) Gravitational force per unit mass is called *gravitational field strength*.

(b)



(b) Since $g \propto \dfrac{1}{r^2}$

$$\frac{g}{g_s} = \frac{\frac{1}{r^2}}{\frac{1}{R^2}} = \frac{R}{r^2}$$

At $r = 2R$, $g = 0.25\ g_s$

At $r = 3R$, $g = 0.11\ g_s$

At $r = 4R$, $g = 0.063\ g_s$

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(c) (i) Since gravitational fields of Earth and Moon are equal in magnitude and opposite in direction. So resultant field is zero at a point in between them.

(ii) Let the distance from centre of Earth where the Gravitational field strength is zero be x.

$$g_E = g_M$$

$$\frac{GM_E}{x^2} = \frac{GM_M}{(60R_E - x)^2}$$

$$\left(\frac{x}{60R_E - x}\right)^2 = \frac{M_E}{M_M}$$

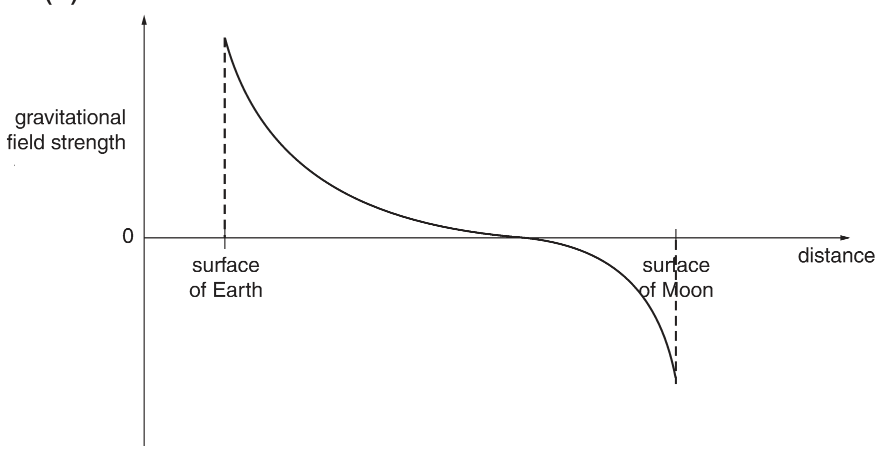
$$\frac{x}{60R_E - x} = \left(\frac{6.00\times10^{24}}{7.40\times10^{22}}\right)^{\frac{1}{2}}$$

$$\frac{x}{60R_E - x} = (81.0)^{\frac{1}{2}}$$

$$540R_E = 10x \quad \Rightarrow \quad x = 54$$

$$\therefore \quad \text{distance} = 54\ R_E$$

(iii)



(c) (iii) — Gravitational field strength of Earth is greater than Moon due to its greater mass.
— Gravitational field strength is a vector quantity and can be +ve or –ve.

Question 3

(a) State what is meant by a *field of force*. [1]

(b) Gravitational fields and electric fields are two examples of fields of force. State one similarity and one difference between these two fields of force. [3]

(c) Two protons are isolated in space. Their centres are separated by a distance R.
Each proton may be considered to be a point mass with point charge. Determine the magnitude of the ratio

$$\frac{\text{force between protons due to electric field}}{\text{force between protons due to gravitational field}}.$$ [3]

[J11/P4/Q1]

Suggested Solution:

(a) Region or space around an object or a charge where another object or charge experience a force.

(b) similarity: Both follow the inverse-square law of force i.e. $F \propto \dfrac{1}{r^2}$

difference: Gravitational force is always attractive while elastic force is either attractive or repulsive.

(c)
$$\frac{F_E}{F_G} = \frac{\dfrac{1}{4\pi\varepsilon_o}\dfrac{Q_P^2}{r^2}}{G\dfrac{m_P^2}{r^2}}$$
$$= \frac{Q_P^2}{4\pi\varepsilon_o G m_P^2}$$
$$= \frac{(1.60\times10^{-19})^2}{4(3.14)(8.85\times10^{-12})(6.67\times10^{-11})(1.67\times10^{-27})^2} = 1.2\times10^{36}$$

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(b) • In both fields potential $\propto \dfrac{1}{r}$

• Direction of force is always along the field lines.

Question 4

(a) A moon is in a circular orbit of radius r about a planet. The angular speed of the moon in its orbit is ω. The planet and its moon may be considered to be point masses that are isolated in space.

Show that r and ω are related by the expression

$$r^3\omega^2 = \text{constant}.$$

Explain your working. [3]

(b) Phobos and Deimos are moons that are in circular orbits about the planet Mars.
Data for Phobos and Deimos are shown in Fig. 1.1.

moon	radius of orbit / m	period of rotation about Mars / hours
Phobos	9.39×10^6	7.65
Deimos	1.99×10^7	

Fig. 1.1

(i) Use data from Fig. 1.1 to determine

1. the mass of Mars, [3]

2. the period of Deimos in its orbit about Mars. [3]

(ii) The period of rotation of Mars about its axis is 24.6 hours.
Deimos is in an equatorial orbit, orbiting in the same direction as the spin of Mars about its axis.
Use your answer in **(i)** to comment on the orbit of Deimos. [1]

[N11/P4/Q1]

Suggested Solution:

(a) Gravitational pull of planet provides Centripetal force to moon.

$$\frac{GMm}{r^2} = mr\omega^2$$
$$GM = r^3\omega^2$$
$$r^3\omega^2 = \text{constant}$$

(a) M – Mass of Planet
m – Mass of moon

(b) (i) 1. $GM = r^3\omega^2$

$$GM = r^3\left(\frac{2\pi}{T}\right)^2$$

$$M = \frac{4\pi^2 r^3}{GT^2}$$

$$M = \frac{4(3.14)^2(9.39\times10^6)^3}{(6.67\times10^{-11})(7.65\times60\times60)^2} = 6.45\times10^{23}\text{ kg}$$

2. $GM = r^3\omega^2$

$$\omega = \sqrt{\frac{GM}{r^3}}$$

$$\omega = \sqrt{\frac{(6.67\times10^{-11})(6.45\times10^{23})}{(1.99\times10^7)^3}} = 7.39\times10^{-5}\text{rad s}^{-1}$$

Since, $\omega = \frac{2\pi}{T}$

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2(3.14)}{7.39\times10^{-5}} = 8.5\times10^4\text{ s} = \frac{8.5\times10^4}{60\times60} = 23.6\text{ hours}$$

(ii) Deimos is in 'geostationary' orbit about Mars due to nearly same time periods.

Question 5

(a) State Newton's law of gravitation. [2]

(b) The Earth and the Moon may be considered to be isolated in space with their masses concentrated at their centres.

The orbit of the Moon around the Earth is circular with a radius of 3.84×10^5 km. The period of the orbit is 27.3 days.

Show that

(i) the angular speed of the Moon in its orbit around the Earth is 2.66×10^{-6} rad s^{-1}, [1]

(ii) the mass of the Earth is 6.0×10^{24} kg. [2]

(c) The mass of the Moon is 7.4×10^{22} kg.

(i) Using data from **(b)**, determine the gravitational force between the Earth and the Moon. [2]

(ii) Tidal action on the Earth's surface causes the radius of the orbit of the Moon to increase by 4.0 cm each year.

Use your answer in **(i)** to determine the change, in one year, of the gravitational potential energy of the Moon. Explain your working. [3]

[J12/P4/Q1]

Suggested Solution:

(a) The force of attraction between two bodies is directly proportional to the product of their masses and inversely proportional to the square of separation between their centres.

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(b) (i) $\omega = \frac{2\pi}{T}$

$= \frac{2(3.14)}{27.3 \times 24 \times 60 \times 60} = 2.66 \times 10^{-6} \text{rad s}^{-1}$

(ii) Gravitational pull of Earth provides centripetal force to Moon

$F_G = F_c$

$\frac{GMm}{r^2} = m r \omega^2$

$M = \frac{r^3 \omega^2}{G}$

$= \frac{(3.84 \times 10^8)^3 (2.66 \times 10^{-6})^2}{6.67 \times 10^{-11}} = 6.0 \times 10^{24} \text{ kg}$

(b) (ii) Astronomical distances are frequently given in km, whereas the formulae are appropriate to distances in metres.

(c) (i) $F = \frac{GMm}{r^2}$

$= \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})(7.4 \times 10^{22})}{(3.84 \times 10^8)^2} = 2.0 \times 10^{20} \text{ N}$

(ii) $\Delta E_p = -\frac{GMm}{r+h} - \left(-\frac{GMm}{r}\right)$

$= GMm\left(\frac{1}{r} - \frac{1}{r+h}\right)$

$= (6.67 \times 10^{-11})(6.0 \times 10^{24})(7.4 \times 10^{22})\left(\frac{1}{3.84 \times 10^8} - \frac{1}{3.84 \times 10^8 + 4.0 \times 10^{-2}}\right)$

$= 8.0 \times 10^{18} \text{ J}$

(c) (ii) Since x is very much smaller than the distance r, so force could be considered as being constant.

So,

$\Delta E_p = (F)(x)$

$= (2.0 \times 10^{20})(4.0 \times 10^{-2})$

$= 8.0 \times 10^{18} \text{ J}$

Question 6

(a) State Newton's law of gravitation. [2]

(b) A satellite of mass m is in a circular orbit of radius r about a planet of mass M.
For this planet, the product GM is 4.00×10^{14} N m^2 kg^{-1}, where G is the gravitational constant.
The planet may be assumed to be isolated in space.

(i) By considering the gravitational force on the satellite and the centripetal force, show that the kinetic energy E_K of the satellite is given by the expression

$$E_K = \frac{GMm}{2r}.$$ [2]

(ii) The satellite has mass 620 kg and is initially in a circular orbit of radius 7.34×10^6 m, as illustrated in Fig. 1.1.

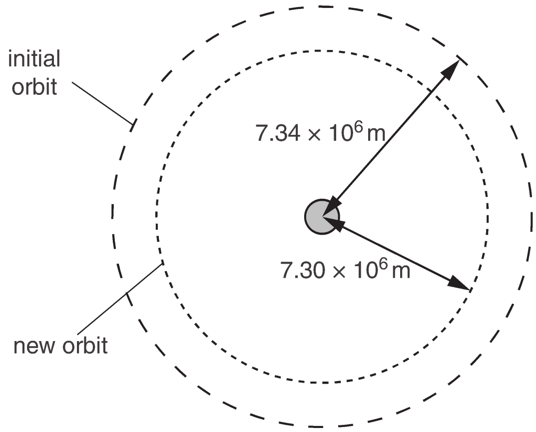

Fig. 1.1 (not to scale)

Resistive forces cause the satellite to move into a new orbit of radius 7.30×10^6 m.
Determine, for the satellite, the change in

1. kinetic energy, [2]

2. gravitational potential energy. [2]

(iii) Use your answers in **(ii)** to explain whether the linear speed of the satellite increases, decreases or remains unchanged when the radius of the orbit decreases. [2]

[N12/P4/Q1]

Suggested Solution:

(a) Force of attraction between two bodies is directly proportional to product of their masses and inversely proportional to the square of separation between their centres.

(b) (i) Gravitational force provides centripetal force

$$\therefore \quad F_G = F_C$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$mv^2 = \frac{GMm}{r}$$

divide both sides by 2,

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

or $$E_K = \frac{GMm}{2r}$$

(ii) 1. $$\Delta E_K = \frac{GMm}{2r_1} - \frac{GMm}{2r_2}$$

$$= \frac{GMm}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{(4.00\times10^{14})(620)}{2}\left(\frac{1}{7.30\times10^6} - \frac{1}{7.34\times10^6}\right)$$

$$= 9.26\times10^7 \text{ J}$$

$\therefore$ change in kinetic energy $= 9.26\times10^7$ J

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2. $$\Delta E_P = -\frac{GMm}{r_1} - \left(-\frac{GMm}{r_2}\right)$$

$$= GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$= (4.00\times10^{14})(620)\left(\frac{1}{7.34\times10^6} - \frac{1}{7.30\times10^6}\right) = -1.85\times10^8 \text{ J}$$

$\therefore$ change in potential energy $= -1.85\times10^8$ J

(iii) Since change in kinetic energy is positive for a constant mass, so speed increases.

Question 7

(a) Explain what is meant by a *geostationary orbit*. [3]

(b) A satellite of mass m is in a circular orbit about a planet.
The mass M of the planet may be considered to be concentrated at its centre.
Show that the radius R of the orbit of the satellite is given by the expression

$$R^3 = \left(\frac{GMT^2}{4\pi^2}\right)$$

where T is the period of the orbit of the satellite and G is the gravitational constant. Explain your working. [4]

(c) The Earth has mass 6.0×10^{24} kg. Use the expression given in **(b)** to determine the radius of the geostationary orbit about the Earth. [3]

[J13/P4/Q1]

Suggested Solution:

(a) Equatorial path at a fixed distance from center of earth along which geostationary satellite revolve in the same direction and rate (24 hours) as that of spinning of earth.

(b) Gravitational pull of planet provides centripetal force to satellite.

$$F_G = F_C$$

$$\frac{GMm}{R^2} = mRw^2$$

$$\frac{GM}{R^2} = R\left(\frac{2\pi}{T}\right)^2 \Rightarrow R^3 = \frac{GMT^2}{4\pi^2}$$

(c) $$R = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}}$$

$$R = \left(\frac{(6.67\times10^{-11})(6.0\times10^{24})(24\times60\times60)^2}{4(3.14)^2}\right)^{\frac{1}{3}}$$

$$R = \left(7.58\times10^{22}\right)^{\frac{1}{3}}$$

$$R = 4.23\times10^7 \text{ m}$$

Question 20

(a) State Newton's law of gravitation. [2]

(b) The astronomer Johannes Kepler showed that the period T of rotation of a planet about the Sun is related to its mean distance *R* from the centre of the Sun by the expression

$$\frac{R^3}{T^2} = k$$

where k is a constant.

Use Newton's law to show that, for planets in circular orbits about the Sun of mass M, the constant k is given by

$$k = \frac{GM}{4\pi^2}$$

where G is the gravitational constant. Explain your working. [4]

(c) A satellite is in a circular orbit about Mars.

The radius of the orbit of the satellite is 4.38×10^6 m. The orbital period is 2.44 hours.

Use the expressions in **(b)** to calculate a value for the mass of Mars. [2]

[N19/P4/Q1]

Suggested Solution:

(a) The force of attraction between two point objects is directly proportional to product of their masses and inversely proportional to the square of separation between their centres.

(b) Gravitational pull of sun provides centripetal force to planet,

$$F_G = F_C$$

$$\frac{GMm}{R^2} = mR\omega^2$$

$$GM = R^3\left(\frac{2\pi}{T}\right)^2 \quad \Rightarrow \quad \frac{R^3}{T^2} = \frac{GM}{4\pi^2}$$

(c) $\dfrac{GM}{4\pi^2} = \dfrac{R^3}{T^2}$

$$M = \frac{4\pi^2 R^3}{GT^2}$$

$$= \frac{4(3.14)^2(4.38\times10^6)^3}{(6.67\times10^{-11})(2.44\times60\times60)^2} = 6.44\times10^{23} \text{ kg}$$

Question 21

(a) Define *gravitational potential* at a point. [2]

(b) An isolated solid sphere of radius r may be assumed to have its mass M concentrated at its centre. The magnitude of the gravitational potential at the surface of the sphere is ϕ.

On Fig. 1.1, show the variation of the gravitational potential with distance d from the centre of the sphere for values of d from $d = r$ to $d = 4r$.

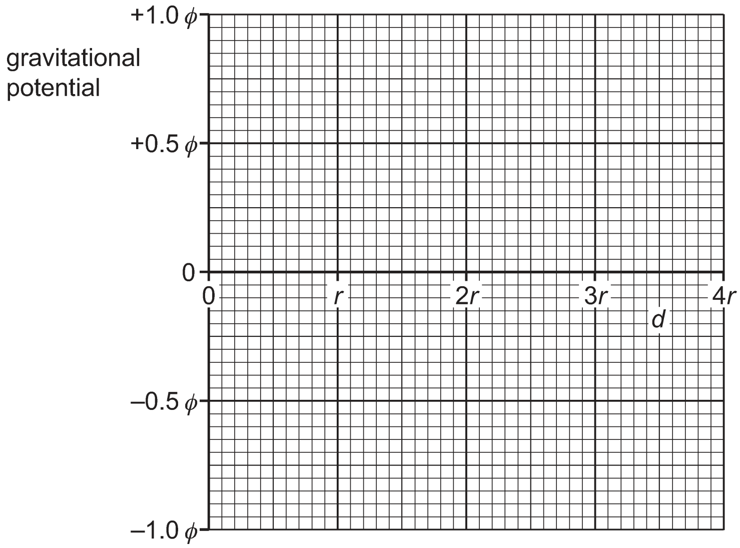


Fig. 1.1 [3]

(c) The sphere in **(b)** is a planet with radius r of 6.4×10^6 m and mass M of 6.0×10^{24} kg. The planet has no atmosphere.

A rock of mass 3.4×10^3 kg moves directly towards the planet. Its distance from the centre of the planet changes from $4r$ to $3r$.

(i) Calculate the change in gravitational potential energy of the rock. [3]

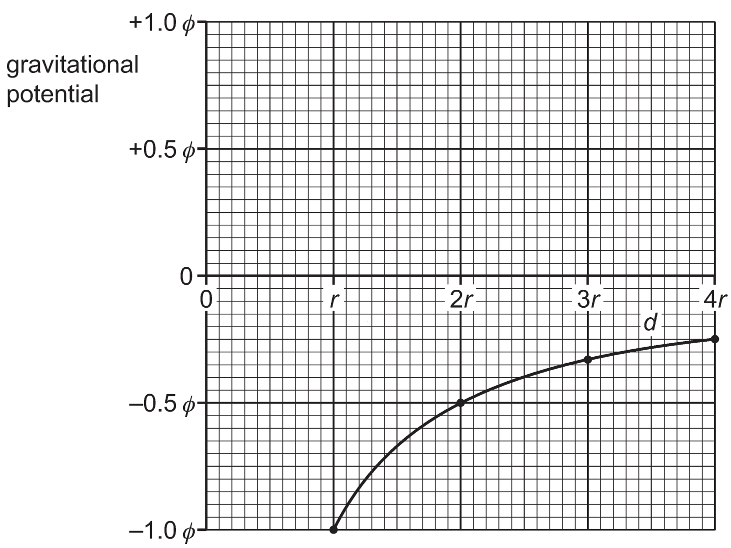
(ii) Explain whether the rock's speed increases, decreases or stays the same. [2]

[J20/P4/Q1]

Suggested Solution:

(a) Amount of work done per unit mass to bring the mass from infinity upto a point in the gravitational field.

(b)



(b) $\phi = -\dfrac{GM}{r}$

r / m	ϕ / J kg^{-1}
r	$-\phi$
$2r$	-0.5ϕ
$3r$	-0.33ϕ
$4r$	-0.25ϕ

(c) (i) $\Delta E_P = -\dfrac{GMm}{4r} - \left(-\dfrac{GMm}{3r}\right)$

$= \dfrac{GMm}{r}\left[\dfrac{1}{3} - \dfrac{1}{4}\right]$

$= \dfrac{GMm}{12r}$

$= \dfrac{(6.67\times10^{-11})(6.0\times10^{24})(3.4\times10^{3})}{12(6.4\times10^{6})}$

$\Delta E_P = 1.77\times10^{10}$ J

(ii) Attractive pull of planet loses potential energy of rock and its kinetic energy and hence speed increases.

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(c) (ii) There is no energy loss as planet has no atmosphere so potential energy changes to kinetic energy.

Question 22

(a) Define *gravitational potential* at a point. [2]

(b) The Earth may be considered to be a uniform sphere of radius 6.4×10^6 m with its mass of 6.0×10^{24} kg concentrated at its centre.

A satellite of mass 2.4×10^3 kg is launched from the Equator. It is placed in an equatorial orbit at a height of 5.6×10^6 m above the Earth's surface.

(i) Calculate the change ΔE_P in gravitational potential energy of the satellite for its movement from the surface of the Earth to its position in the equatorial orbit. [3]

(ii) Determine the speed of the satellite when in orbit. [3]

(c) Before the satellite in **(b)** is launched, its speed at the Equator due to the Earth's rotation is 470 m s^{-1}.

Suggest why the energy required to launch the satellite depends on whether the satellite, in its orbit, is travelling from west to east or from east to west. [1]

[N20/P4/Q1]

Suggested Solution:

(a) Amount of work done per unit mass to bring it from infinity upto a point in the gravitational field.

(b) (i) $\Delta E_P = GMm\left[\dfrac{1}{R} - \dfrac{1}{r}\right]$

$= (6.67\times10^{-11})(6.0\times10^{24})(2.4\times10^{3})\left[\dfrac{1}{6.4\times10^{6}} - \dfrac{1}{(6.4+5.6)10^{6}}\right]$

$= 7.0\times10^{10}$ J

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(ii) Gravitational force provides centripetal force

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{(6.67\times10^{-11})(6.0\times10^{24})}{(6.4+5.6)\times10^6}}$$

$$v = 5.77\times10^3 \text{ m s}^{-1}$$

(c) Smaller gain in energy if orbit is in same direction as rotation of Earth i.e. from West to East.

(c) Satellite already moving at launch in the direction of rotation of Earth i.e. from West to East.

Question 23

(a) Define *gravitational field strength*. [1]

(b) An isolated planet is a uniform sphere of radius 3.39×10^6 m. Its mass of 6.42×10^{23} kg may be considered to be a point mass concentrated at its centre. The planet rotates about its axis with a period of 24.6 hours.

For an object resting on the surface of the planet at the equator, calculate, to three significant figures:

(i) the gravitational field strength [2]

(ii) the centripetal acceleration [2]

(iii) the force per unit mass exerted on the object by the surface of the planet. [1]

[J21/P4/Q1]

Suggested Solution:

(a) Attractive force per unit mass of an object.

(a) $g = \frac{F_G}{m}$

(b) (i) $g = \frac{GM}{r^2}$

$$g = \frac{(6.67\times10^{-11})(6.42\times10^{23})}{(3.39\times10^6)^2}$$

$$g = 3.73 \text{ N kg}^{-1}$$

(ii) $a = r\omega^2$

$$a = r\left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2 r}{T^2}$$

$$a = \frac{4(3.14)^2(3.39\times10^6)}{(24.6\times60\times60)^2}$$

$$a = 0.0171 \text{ m s}^{-2}$$

(iii) At the equator,

$F_G = F_C + W$

$\Rightarrow \; F_G - F_C = R$

$\Rightarrow \; mg - ma = R \; \Rightarrow \; \dfrac{R}{m} = g - a$

$\therefore$ Force per unit mass $= g - a$

$= 3.73 - 0.0171 = 3.71\ \text{N kg}^{-1}$

Learning CORNER

Question 24

(a) State the relationship between gravitational potential and gravitational field strength. [2]

(b) A moon of mass M and radius R orbits a planet of mass $3M$ and radius $2R$. At a particular time, the distance between their centres is D, as shown in Fig. 2.1.

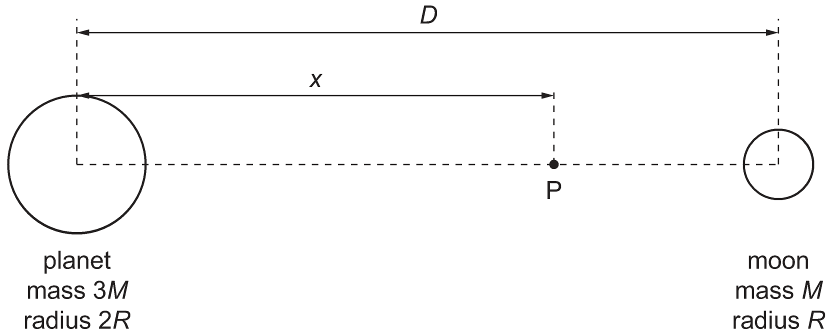


Fig. 2.1

Point P is a point along the line between the centres of the planet and the moon, at a variable distance x from the centre of the planet.

The variation with x of the gravitational potential ϕ at point P, for points between the planet and the moon, is shown in Fig. 2.2.

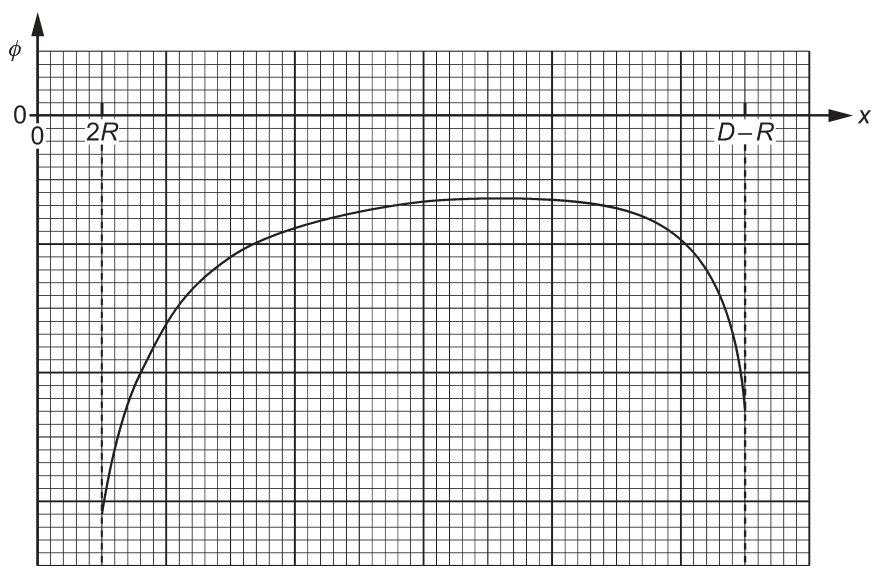


Fig. 2.2

(i) Explain why ϕ is negative throughout the entire range $x = 2R$ to $x = D - R$. [3]

(ii) One of the features of Fig. 2.2 is that ϕ is negative throughout. Describe **two** other features of Fig. 2.2. [2]

(iii) On Fig. 2.3, sketch the variation with x of the gravitational field strength g at point P between $x = 2R$ and $x = D - R$.

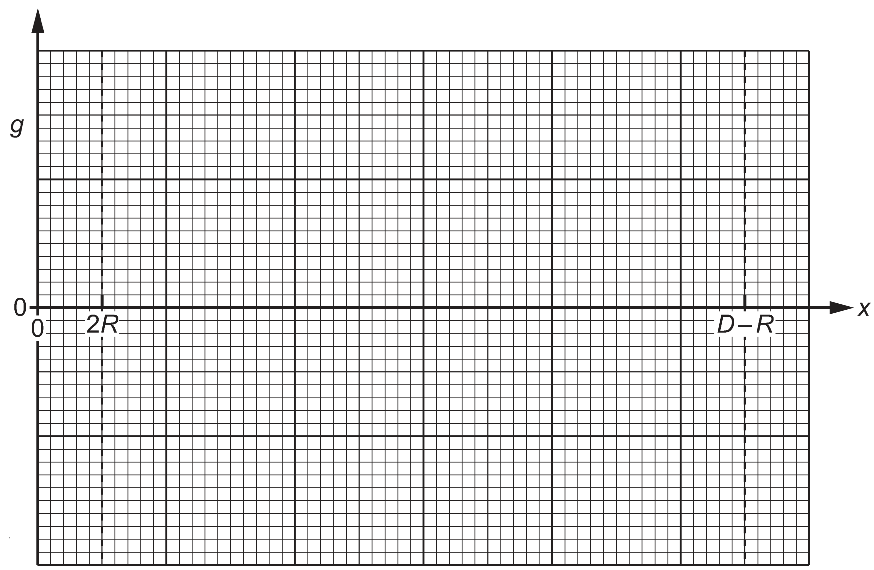


Fig. 2.3 [3]

[N21/P4/Q2]

Suggested Solution:

(a) Gravitational field strength is equal to negative potential gradient. Negative sign is due to attractive gravitational field. Energy is also required to move unit mass to infinity against the field.

(b) (i) Gravitational force is attractive and potential at infinity is zero. So potential energy of test mass decreases as it approaches from infinity.

(ii) 1. Potential at the surface of planet is lesser than at the surface of moon.

2. Point of maximum potential is near to moon and away from planet.

Learning CORNER

(a) $g = \frac{GM}{r^2}$

Also, $\phi = -\frac{GM}{r}$

So, $g = -\frac{\phi}{r}$

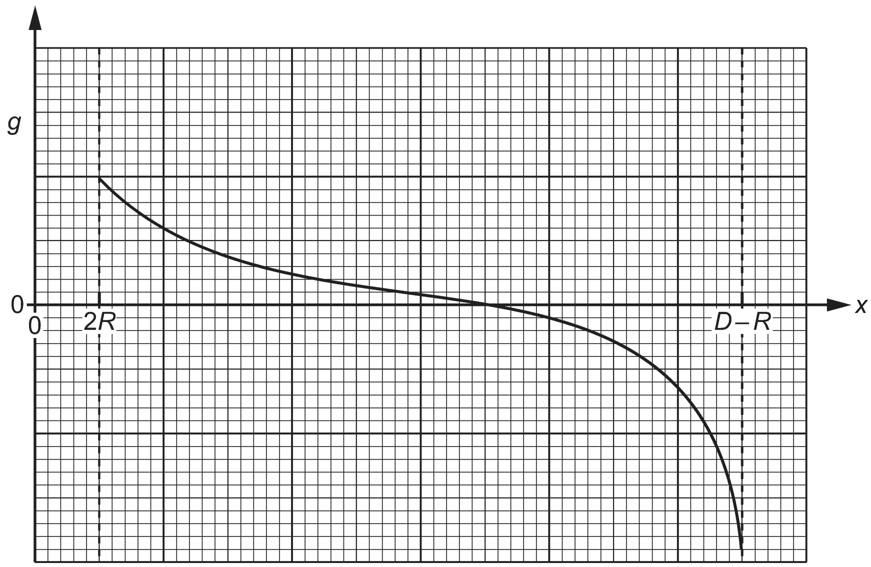
(b) (ii) Also potential gradient at the surface of planet is less than at surface of moon.

(iii) $g = -\frac{\phi}{m}$

Gravitational field strength is zero in the region between planet and moon, so its direction is changed at this point.

(iii)

Learning CORNER



Question 25

(a) (i) Define gravitational potential at a point. [2]

(ii) Starting from the equation for the gravitational potential due to a point mass, show that the gravitational potential energy E_P of a point mass m at a distance r from another point mass M is given by

$$E_P = -\frac{GMm}{r}$$

where G is the gravitational constant. [1]

(b) Fig. 1.1 shows the path of a comet of mass 2.20×10^{14} kg as it passes around a star of mass 1.99×10^{30} kg.

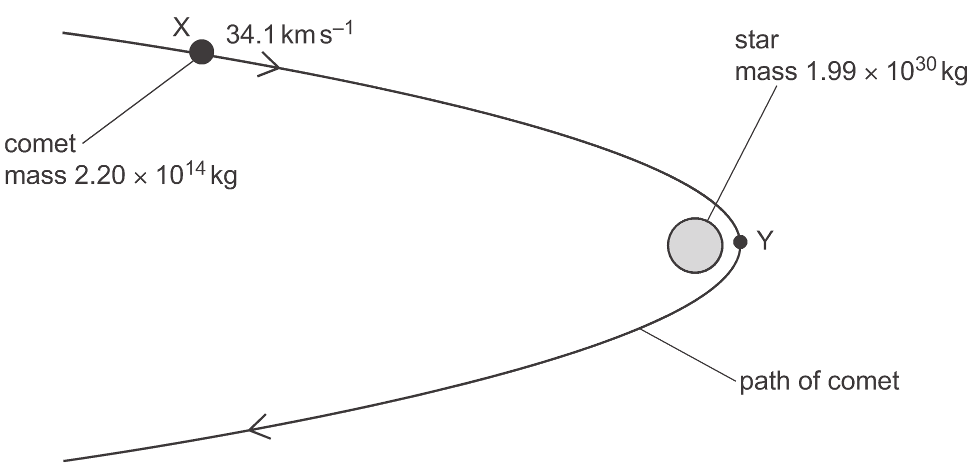


Fig. 1.1 (not to scale)

At point X, the comet is 8.44×10^{11} m from the centre of the star and is moving at a speed of 34.1 km s^{-1}.

Learning CORNER

At point Y, the comet passes its point of closest approach to the star. At this point, the comet is a distance of 6.38×10^{10} m from the centre of the star.

Both the comet and the star can be considered as point masses at their centres.

(i) Calculate the magnitude of the change in the gravitational potential energy ΔE_P of the comet as it moves from position X to position Y. [2]

(ii) State, with a reason, whether the change in gravitational potential energy in **(b)(i)** is an increase or a decrease. [1]

(iii) Use your answer in **(b)(i)** to determine the speed, in km s^{-1}, of the comet at point Y. [3]

(c) A second comet passes point X with the same speed as the comet in **(b)** and travelling in the same direction. This comet is gradually losing mass. The mass of this comet when it passes point X is the same as the mass of the comet in **(b)**.

Suggest, with a reason, how the path of the second comet compares with the path shown in Fig. 1.1. [1]

[J22/P4/Q1]

Suggested Solution:

(a) (i) Amount of work done per unit mass required to bring it from infinity upto a point in the gravitational field.

(a) (i) $\phi = \dfrac{W}{m}$

(ii) $$\phi = \frac{W}{m}$$

$$-\frac{GM}{r} = \frac{W}{m} \Rightarrow W = -\frac{GMm}{r}$$

As work done is stored as gravitational potential energy,

so, $E_p = -\dfrac{GMm}{r}$

(b) (i) $$\Delta E_P = -\frac{GMm}{r_Y} - \left(-\frac{GMm}{r_X}\right)$$

$$= GMm\left[\frac{1}{r_X} - \frac{1}{r_Y}\right]$$

$$= (6.67\times10^{-11})(1.99\times10^{30})(2.20\times10^{14})\left[\frac{1}{8.44\times10^{11}} - \frac{1}{6.38\times10^{10}}\right]$$

$$= (6.67\times10^{-11})(1.99\times10^{30})(2.20\times10^{14})(-1.45\times10^{-11})$$

$$= -4.23\times10^{23}$$

–ve sign also shows the decrease in the potential energy of comet.

$\therefore \quad \Delta E_P = 4.23\times10^{23}$ J

(ii) Since work is done by attractive pull of star, so gravitational potential energy decreases.

(iii)

$$\text{Total energy at X} = \text{Total energy at Y}$$

$$\frac{1}{2}mv_X^2 + \left(-\frac{GMm}{r_X}\right) = \frac{1}{2}mv_Y^2 + \left(-\frac{GMm}{r_Y}\right)$$

$$\frac{GMm}{r_Y} - \frac{GMm}{r_X} + \frac{1}{2}mv_X^2 = \frac{1}{2}mv_Y^2$$

$$\Delta E_P + \frac{1}{2}mv_X^2 = \frac{1}{2}mv_Y^2$$

$$4.23\times10^{23} + \frac{1}{2}(2.20\times10^{14})(34100)^2 = \frac{1}{2}(2.20\times10^{14})(v_Y^2)$$

$$v = 70.8\times10^3 = 70.8\ \text{km s}^{-1}$$

(c) No change of path because both K.E. and G.P.E. include mass which is cancelled out if total energy at X = total energy at Y.

Question 26

(a) Define gravitational field. [1]

(b) A spherical planet can be considered as a point mass at its centre.

(i) On Fig. 1.1, draw gravitational field lines outside the planet to represent the gravitational field due to the planet.

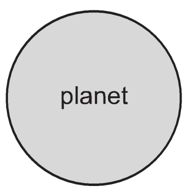


Fig. 1.1

[2]

(ii) A satellite is in a circular orbit around the planet.

Explain, with reference to your answer in **(b)(i)**, why the path of the satellite is circular. [2]

(c) An object rests on the surface of the Earth at the Equator. The radius of the Earth is 6.4×10^6 m.

(i) Determine the centripetal acceleration of the object. [3]

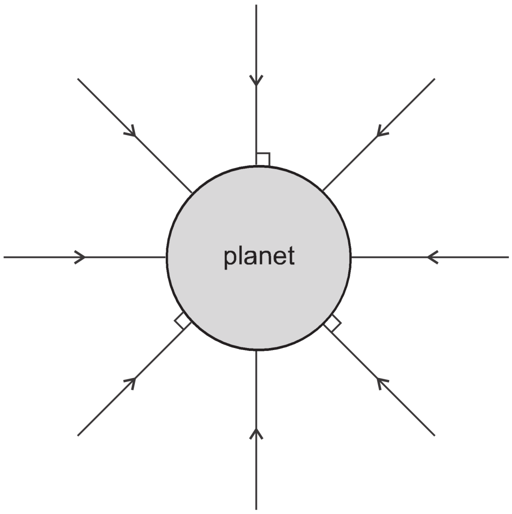
(ii) Describe how the two forces acting on the object give rise to this centripetal acceleration. You may draw a diagram if you wish. [2]

[N22/P4/Q1]

Suggested Solution:

(a) A 3D region or space around a massive object (star, planet etc) in which a unit mass experience an attractive force.

(b) (i)



(ii) Field lines represent force on a satellite perpendicular to its velocity. Therefore gravitational force provides centripetal force to cause centripetal acceleration.

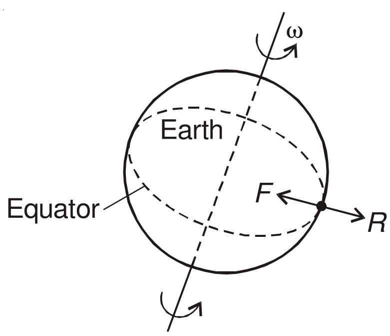
(c) (i) $a = R\omega^2$

$$= R\left(\frac{2\pi}{T}\right)^2$$

$$= \frac{4\pi^2 R}{T^2}$$

$$= \frac{4(3.14)^2(6.4\times10^6)}{(24\times60\times60)^2} = 0.0338\ \text{ms}^{-2}$$

(ii)



F : Gravitaional force towards center of planet

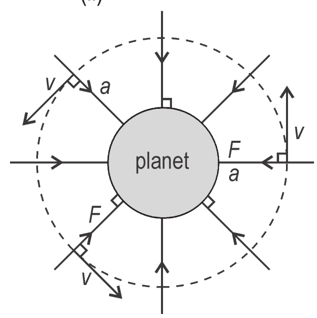
R : Normal reactional contact force

Gravitational pull on object and normal contact forces are in opposite directions but their resultant effect provides centripetal force and hence acceleration towards center of earth.

Learning CORNER

(b) (i) Non-uniform field i.e., gap between adjacent field lines vary.
Each field line is perpendicular to the surface of planet.

(ii)



Topic 10

Magnetic Fields

10.1 Concept of a magnetic field

Candidates should be able to:

1 understand that a magnetic field is an example of a field of force produced either by moving charges or by permanent magnets

2 represent a magnetic field by field lines

10.2 Force on a current-carrying conductor

Candidates should be able to:

1 understand that a force might act on a current-carrying conductor placed in a magnetic field

2 recall and use the equation $F = BIL\sin\theta$ with directions as interpreted by Fleming's left-hand rule

3 define magnetic flux density as the force acting per unit current per unit length on a wire placed at right angles to the magnetic field

10.3 Force on a moving charge

Candidates should be able to:

1 determine the direction of the force on a charge moving in a magnetic field

2 recall and use $F = BQv\sin\theta$

3 understand the origin of the Hall voltage and derive and use the expression $V_H = \frac{BI}{(ntq)}$, where t = thickness

4 understand the use of a Hall probe to measure magnetic flux density

5 describe the motion of a charged particle moving in a uniform magnetic field perpendicular to the direction of motion of the particle

6 explain how electric and magnetic fields can be used in velocity selection

10.4 Magnetic fields due to currents

Candidates should be able to:

1 sketch magnetic field patterns due to the currents in a long straight wire, a flat circular coil and a long solenoid

2 understand that the magnetic field due to the current in a solenoid is increased by a ferrous core

3 explain the origin of the forces between current-carrying conductors and determine the direction of the forces

10.5 Electromagnetic induction

Candidates should be able to:

1 define magnetic flux as the product of the magnetic flux density and the cross-sectional area perpendicular to the direction of the magnetic flux density

2 recall and use $\Phi = BA$

3 understand and use the concept of magnetic flux linkage

4 understand and explain experiments that demonstrate:
- that a changing magnetic flux can induce an e.m.f. in a circuit
- that the induced e.m.f. is in such a direction as to oppose the change producing it
- the factors affecting the magnitude of the induced e.m.f.

5 recall and use Faraday's and Lenz's laws of electromagnetic induction

Learning CORNER

TOPIC 10 *opening ....*

Question 1

(a) Define the *tesla*. [3]

(b) A horseshoe magnet is placed on a balance. A stiff metal wire is clamped horizontally between the poles, as illustrated in Fig. 5.1.

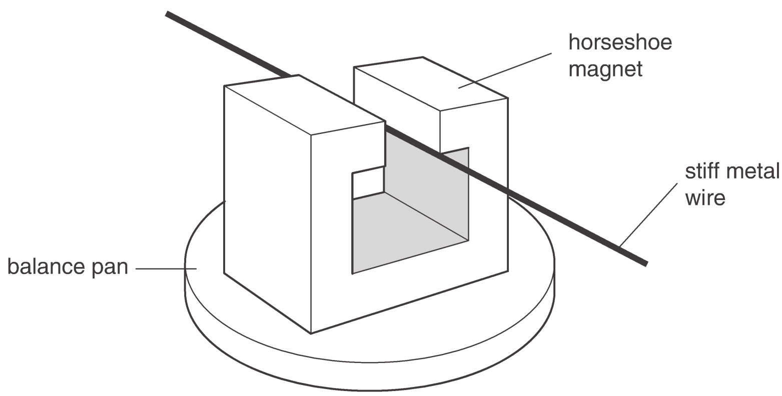


Fig. 5.1

The magnetic flux density in the space between the poles of the magnet is uniform and is zero outside this region.
The length of the metal wire normal to the magnetic field is 6.4 cm.

When a current in the wire is switched on, the reading on the balance increases by 2.4 g. The current in the wire is 5.6 A.

(i) State and explain the direction of the force on the wire due to the current. [3]

(ii) Calculate the magnitude of the magnetic flux density between the poles of the magnet. [2]

(c) A low frequency alternating current is now passed through the wire in **(b)**.
The root-mean-square (r.m.s.) value of the current is 5.6 A.
Describe quantitatively the variation of the reading seen on the balance. [2]

[J12/P4/Q5]

Suggested Solution:

(a) Magnetic flux density is 1T if a force of 1N is experienced by 1m length of a conductor carrying 1A current when placed perpendicular to magnetic field lines.

(a) $F = BIL\sin\theta$

$$B = \frac{F}{IL\sin 90°}$$

$$1\text{ T} = \frac{1\text{N}}{(1\text{ A})(1\text{ m})\sin 90°}$$

(b) (i) An increase in balance reading indicates an increase of downward force on magnet due to current in wire. So wire experienced an upward force as per Newton's third law.

(ii)
$$F = BIL\sin\theta$$
$$mg = BIL\sin 90°$$
$$B = \frac{mg}{IL}$$
$$B = \frac{(2.4\times10^{-3})(9.81)}{(5.6)(6.4\times10^{-2})} = 0.066\text{ T}$$

Learning CORNER

(c) $I_0 = (\sqrt{2})(I_{rms}) \Rightarrow I_0 = (\sqrt{2})(5.6) = 7.92$ A

5.6 A current causes a change in reading = 2.4 g

1A current causes a change in reading $= \dfrac{2.4}{5.6}$

7.92 A current causes a change in reading $= \dfrac{2.4}{5.6} \times 7.92 = 3.4$ g

For both half cycles of AC

Total change in reading = 2(3.4)
= 6.8 g

Question 2

(a) (i) State the condition for a charged particle to experience a force in a magnetic field. [2]

(ii) State an expression for the magnetic force *F* acting on a charged particle in a magnetic field of flux density *B*. Explain any other symbols you use. [2]

(b) A sample of a conductor with rectangular faces is situated in a magnetic field, as shown in Fig. 6.1.

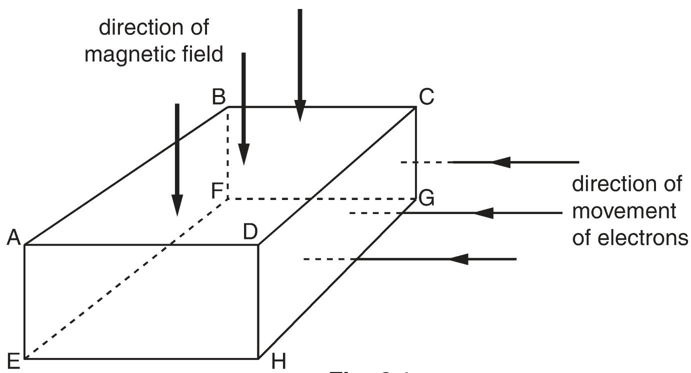


Fig. 6.1

The magnetic field is normal to face ABCD in the downward direction. Electrons enter face CDHG at right-angles to the face. As the electrons pass through the conductor, they experience a force due to the magnetic field.

(i) On Fig. 6.1, shade the face to which the electrons tend to move as a result of this force. [1]

(ii) The movement of the electrons in the magnetic field causes a potential difference between two faces of the conductor.
Using the lettering from Fig. 6.1, state the faces between which this potential difference will occur. [1]

(c) Explain why the potential difference in **(b)** causes an additional force on the moving electrons in the conductor. [2]

[N12/P4/Q6]

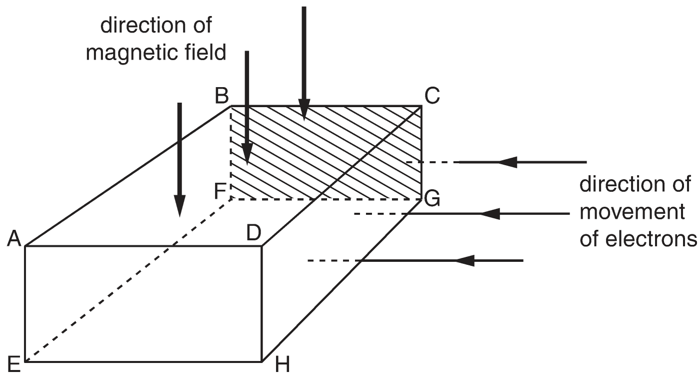
Suggested Solution:

(a) (i) Since $F = Bqv\sin\theta \Rightarrow F = Bqv\sin 90°$
Charge must move ($v \neq 0$) and the velocity or its component must be perpendicular to magnetic field lines.

(ii) $F = Bqv\sin\theta$, where,
q - charge on particle, v - velocity of particle and θ - angle between magnetic flux density and motion (velocity) of the particle.

Learning CORNER

(b) (i)



(b) (i) Fleming's left hand rule is used to determine direction.

(ii) face ADHE and face BCGF.

(c) Potential difference provides an electric field which therefore gives rise to force on electrons by $F = \frac{vq}{d}$.

Question 3

(a) Define the *tesla*. [2]

(b) Two long straight vertical wires X and Y are separated by a distance of 4.5 cm, as illustrated in Fig. 5.1.

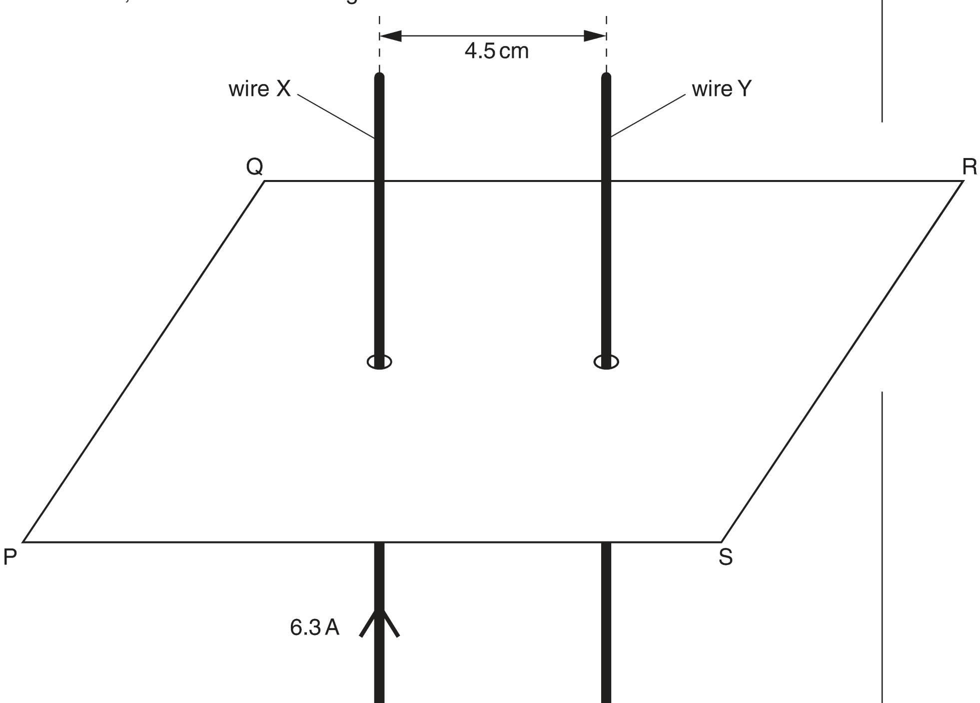


Fig. 5.1

The wires pass through a horizontal card PQRS.
The current in wire X is 6.3 A in the upward direction. Initially, there is no current in wire Y.

(i) On Fig. 5.1, sketch, in the plane PQRS, the magnetic flux pattern due to the current in wire X. Show at least four flux lines. [3]

(ii) The magnetic flux density B at a distance x from a long straight current-carrying wire is given by the expression

$$B = \frac{\mu_0 I}{2\pi x}$$

where I is the current in the wire and μ_0 is the permeability of free space.

Calculate the magnetic flux density at wire Y due to the current in wire X. [2]

(iii) A current of 9.3 A is now switched on in wire Y. Use your answer in **(ii)** to calculate the force per unit length on wire Y. [2]

(c) The currents in the two wires in **(b)(iii)** are not equal.
Explain whether the force per unit length on the two wires will be the same, or different. [2]

[J13/P4/Q5]

Learning CORNER

Suggested Solution:

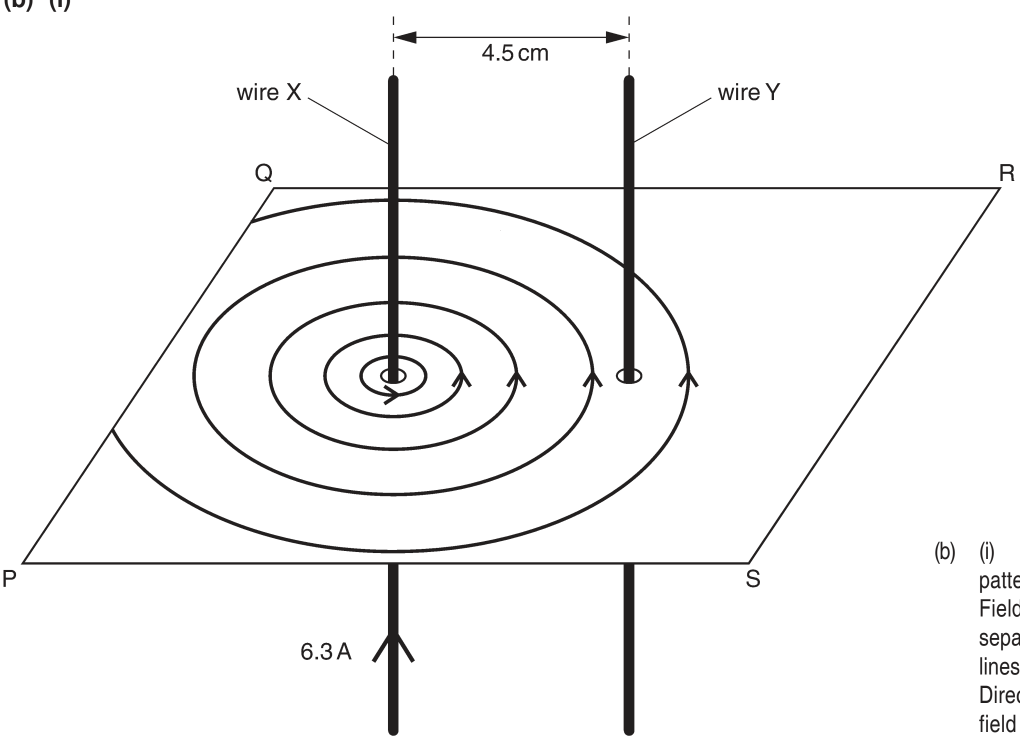
(a) Magnetic flux density is 1 tesla if a force of 1 N is experienced by 1 m length of a conductor carrying a current of 1 A when placed perpendicular to magnetic-field lines.

(a) $F = BIL\sin\theta$

$B = \dfrac{F}{IL\sin\theta}$

$1\text{T} = \dfrac{1\text{N}}{(1\text{A})(1\text{m})\sin 90^\circ}$

(b) (i)



(b) (i) Concentric circular pattern.
Field is non-uniform, so separation between field lines increases.
Direction of magnetic field is obtained by right hand grip rule.

(ii)
$$B_x = \frac{\mu_0 I_x}{2\pi x}$$
$$= \frac{(4\pi\times10^{-7})(6.3)}{2\pi(4.5\times10^{-2})} = 2.8\times10^{-5}$$

$\therefore$ flux density $= 2.8\times10^{-5}$ T

(iii) $F_Y = B_X I_Y L_Y \sin\theta$
$$\frac{F_Y}{L_Y} = (B_X)(I_Y)\sin 90°$$
$$= (2.8\times10^{-5})(9.3)$$
$$= 2.60\times10^{-4}$$

$\therefore$ force per unit length $= 2.60\times10^{-4}$ Nm^{-1}

(c) Force between two parallel conductors depends directly on product of their currents. So force per unit length on both wires is equal in magnitude but opposite in directions by Newton's third law.

Question 4

(a) An incomplete diagram for the magnetic flux pattern due to a current-carrying solenoid is illustrated in Fig. 5.1.

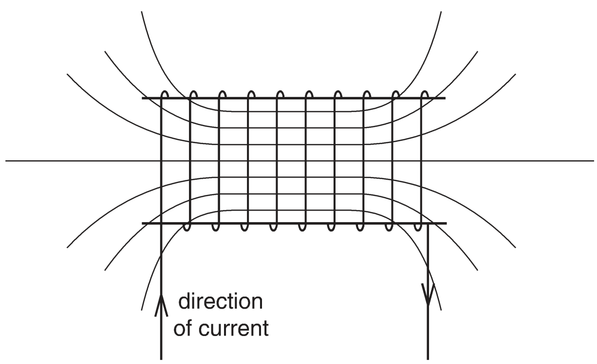


Fig. 5.1

(i) On Fig. 5.1, draw arrows on the field lines to show the direction of the magnetic field. [1]

(ii) State the feature of Fig. 5.1 that indicates that the magnetic field strength at each end of the solenoid is less than that at the centre. [1]

(b) A Hall probe is placed near one end of the solenoid in **(a)**, as shown in Fig. 5.2.

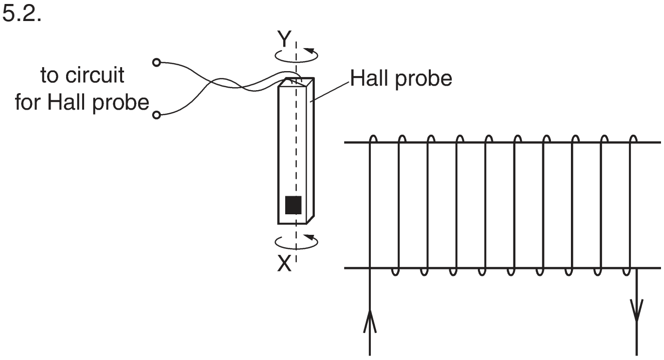


Fig. 5.2

The Hall probe is rotated about the axis XY. State and explain why the magnitude of the Hall voltage varies. [2]

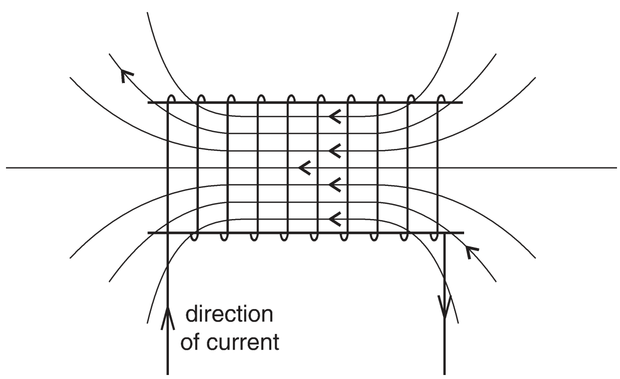
(c) (i) State Faraday's law of electromagnetic induction. [2]

(ii) The Hall probe in **(b)** is replaced by a small coil of wire connected to a sensitive voltmeter.
State three different ways in which an e.m.f. may be induced in the coil. [3]

[N13/P4/Q5]

Suggested Solution:

(a) (i)



(ii) Separations between field lines are larger at ends than at centre.

(b) Hall voltage developed depends upon an angle between plane of probe and the magnetic field lines. Hall p.d. is maximum if plane of probe is perpendicular to field lines and reduces to zero when field lines are parallel to the plane of probe.

(c) (i) Rate of change of magnetic flux linkage is directly proportional to the e.m.f. induced.

(ii) 1. Either rotate coil or move it towards or away from solenoid.
2. Vary current in solenoid.
3. Place an iron core into the solenoid.

Question 5

A charged particle of mass *m* and charge $-q$ is travelling through a vacuum at constant speed *v*.
It enters a uniform magnetic field of flux density *B*. The initial angle between the direction of motion of the particle and the direction of the magnetic field is 90°.

(a) Explain why the path of the particle in the magnetic field is the arc of a circle. [3]

(b) The radius of the arc in **(a)** is *r*.
Show that the ratio $\frac{q}{m}$ for the particle is given by the expression

$$\frac{q}{m} = \frac{v}{Br}.$$ [1]

Learning CORNER

(a) North pole is at left side by Right Hand Rule. Field lines are directed from South to North inside solenoid.

(c) (i) $E \propto \frac{\Delta(N\phi)}{\Delta t}$

Where $\Delta(N\phi)$ is the change in the flux linkage in a time Δt. The constant of proportionality is equal to −1 by lenz's law

$E = \frac{-\Delta(N\phi)}{\Delta t}$.

(ii) For a coil,

$\Delta\phi = \Delta(NBA)$
but $B = \mu_0 NI$
$\Delta\phi = \Delta[N(\mu_0 NI)A]$

So magnetic flux linkage changes if,

— Current in coil changes

— Magnetic flux density changes i.e. place an iron core in solenoid or vary separation between opposite poles of magnets.

Learning CORNER

(c) The initial speed v of the particle is 2.0×10^7 m s^{-1}. The magnetic flux density B is 2.5×10^{-3} T.
The radius r of the arc in the magnetic field is 4.5 cm.

(i) Use these data to calculate the ratio $\dfrac{q}{m}$. [2]

(ii) The path of the negatively-charged particle before it enters the magnetic field is shown in Fig. 6.1.

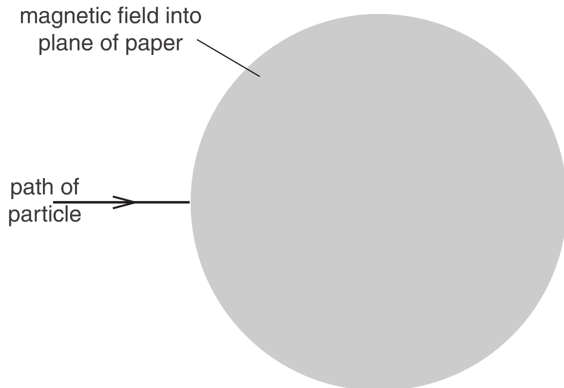


Fig. 6.1

The direction of the magnetic field is into the plane of the paper.
On Fig. 6.1, sketch the path of the particle in the magnetic field and as it emerges from the field. [2]

[N13/P4/Q6]

Suggested Solution:

(a) Magnitude of force on moving charged particle is constant due to constant charge, speed and perpendicular uniform magnetic flux density by $F = Bqv \sin 90°$. Also the direction of force is normal to both field lines and initial direction of motion of charged particle. Therefore this force provides centripetal force to trace a curved path.

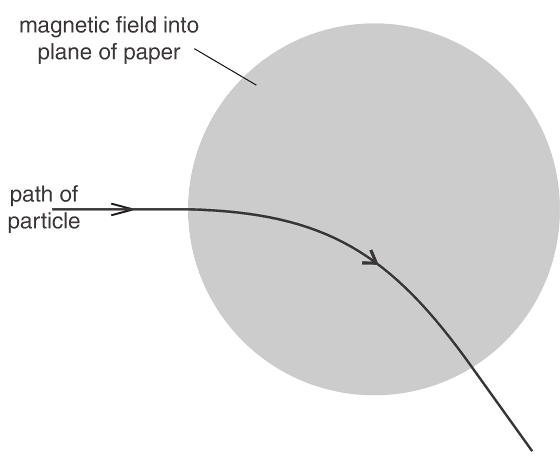
(b) Magnetic force provides centripetal force

$$Bqv\sin 90° = \frac{mv^2}{r}$$

$$Bq = \frac{mv}{r} \quad \Rightarrow \quad \frac{q}{m} = \frac{v}{Br}$$

(c) (i) $\dfrac{q}{m} = \dfrac{2.0\times10^{7}}{(2.5\times10^{-3})(4.5\times10^{-2})} = 1.78\times10^{11}\ \text{Ckg}^{-1}$

(ii)



Learning CORNER

Question 16

(a) State Faraday's law of electromagnetic induction. [2]

(b) An alternating current is passed through an air-cored solenoid.
An iron core is inserted into the solenoid and then held stationary within the solenoid. The current in the solenoid is now smaller.

Explain why the root-mean-square (r.m.s.) value of the current in the solenoid is reduced as a result of inserting the core. [3]

(c) Practical transformers are very efficient. However, there are some power losses.

State two sources of power loss within a transformer. [2]

[N16/P4/Q11]

Suggested Solution:

(a) Rate of change of magnetic flux linkage is directly proportional to the e.m.f induced.

(b) Iron core provides larger change of flux linkage per unit time for same current. So greater is the induced e.m.f in the solenoid. This induced e.m.f opposes the applied e.m.f and so the current reduces.

(c) 1. Heat energy is dissipated in the iron core due to Eddy currents.
2. Heat energy is also dissipated due to current because of resistance of coils.

Question 17

A bar magnet of mass 250 g is suspended from the free end of a spring, as illustrated in Fig. 3.1.

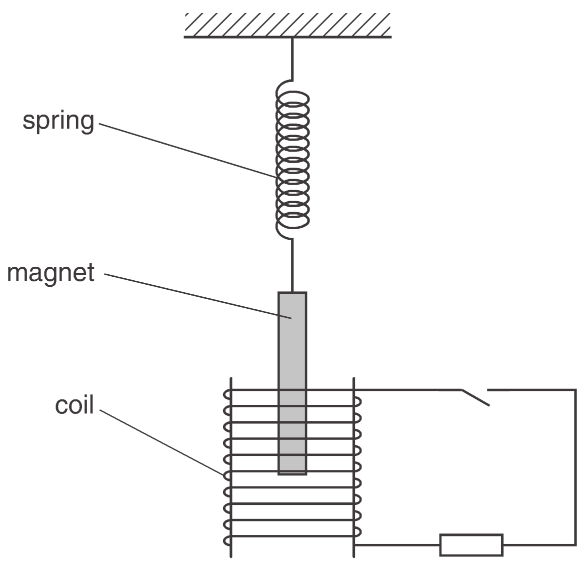


Fig. 3.1

The magnet hangs so that one pole is near the centre of a coil of wire.

The coil is connected in series with a resistor and a switch. The switch is open.

The magnet is displaced vertically and then allowed to oscillate with one pole remaining inside the coil. The other pole remains outside the coil.

Learning CORNER

At time t = 0, the magnet is oscillating freely as it passes through its equilibrium position. At time t = 6.0 s, the switch in the circuit is closed.

The variation with time t of the vertical displacement y of the magnet is shown in Fig. 3.2.

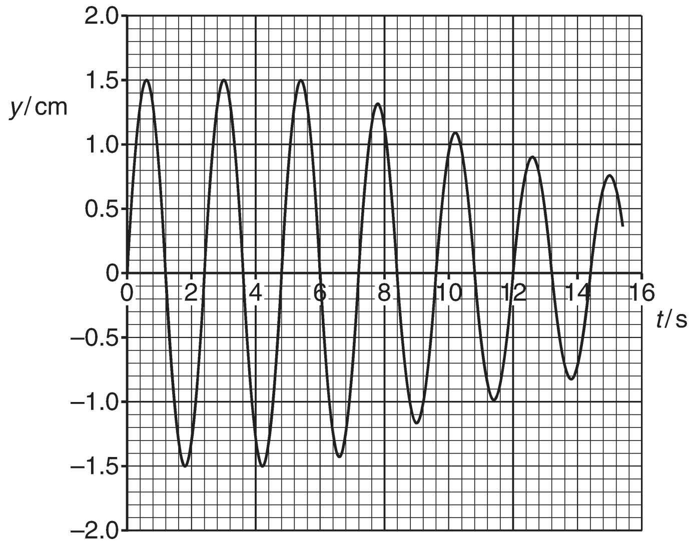


Fig. 3.2

(a) For the oscillating magnet, use data from Fig. 3.2 to calculate, to two significant figures,

(i) the frequency f, [2]

(ii) the energy of the oscillations during the time t = 0 to time t = 6.0 s. [3]

(b) (i) State Faraday's law of electromagnetic induction. [2]

(ii) Use Faraday's law and energy conservation to explain why the amplitude of the oscillations of the magnet reduces after time t = 6.0 s. [3]

[J17/P4/Q3]

Suggested Solution:

(a) (i) 2.5 waves complete in 6.0 s

1 wave complete in $\dfrac{6.0}{2.5} = 2.4$ s

frequency, $f = \dfrac{1}{T} = \dfrac{1}{2.4} = 0.42$ Hz

(ii) Energy, $E_T = \dfrac{1}{2} m\omega^2 y_o^2$

$$= \frac{1}{2} m(2\pi f)^2 y_o^2$$
$$= 2\pi^2 m f^2 y_o^2$$
$$= 2(3.14)^2 (250\times10^{-3})(0.42)^2 (1.5\times10^{-2})^2$$
$$= 1.96\times10^{-4} \approx 2.0\times10^{-4} \text{ J}$$

(b) (i) Rate of change of magnetic flux linkage is directly proportional to the e.m.f. induced.

Learning CORNER

(ii) Motion of magnet changes the magnetic flux linkage per unit time with coil to induce e.m.f. Induced current flows through the coil as circuit is complete and dissipate heat energy in the resistor by $E = I^2Rt$. This energy is derived from the energy of vibrating magnet so its energy decreases and so is its amplitude by $E_T = \frac{1}{2} m\omega^2 y_o^2 \Rightarrow E_T \propto y_o^2$.

Question 18

A Hall probe is placed near to one end of a current-carrying solenoid, as shown in Fig. 9.1.

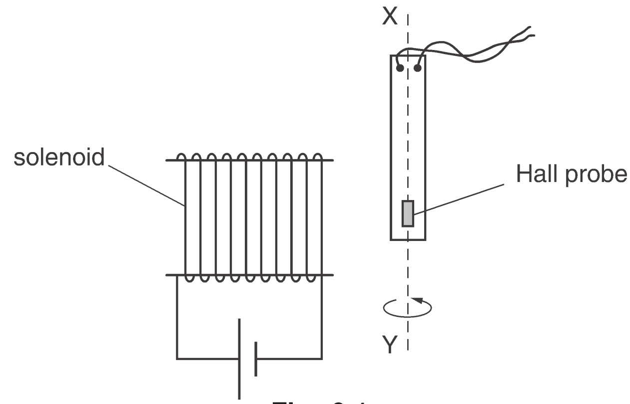


Fig. 9.1

The probe is rotated about the axis XY and is then held in a position so that the Hall voltage is maximum.

(a) Explain why

(i) a Hall probe is made from a *thin slice* of material, [2]

(ii) in order for consistent measurements of magnetic flux density to be made, the current in the probe must be constant. [1]

(b) The probe is now rotated through an angle of 360° about the axis XY.

At angle $\theta = 0$, the Hall voltage V_H has maximum value V_{MAX}.

On Fig. 9.2, sketch the variation with angle θ of the Hall voltage V_H for one complete revolution of the probe about axis XY.

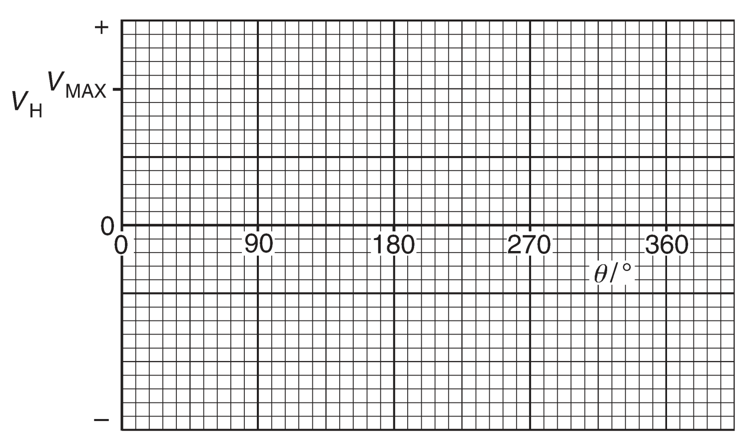


Fig. 9.2 [3]

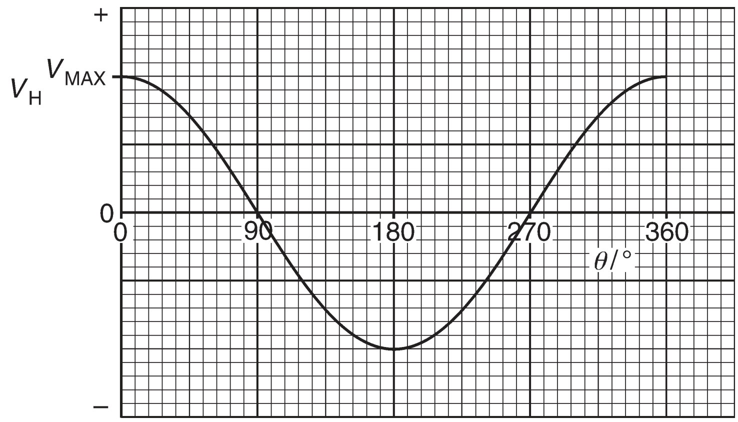
[J17/P4/Q9]

Suggested Solution:

(a) (i) Hall voltage depends inversely on thickness of slice. So greater Hall voltage is developed if slice is thin.

(ii) Hall voltage depends upon current in the slice.

(b)



Learning CORNER

(a) (i) $V_H = \dfrac{BI}{ntq}$

$V_H \propto \dfrac{1}{t}$

Question 19

A thin slice of conducting material is placed normal to a uniform magnetic field, as shown in Fig. 8.1.

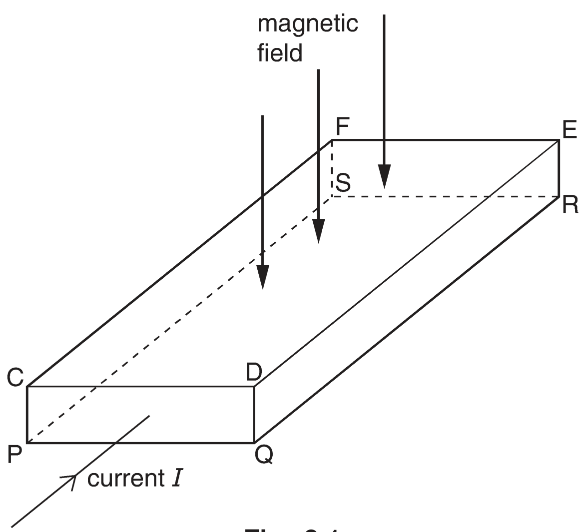


Fig. 8.1

The magnetic field is normal to face CDEF and to face PQRS.

The current I in the slice is normal to the faces CDQP and FERS.

A potential difference, the Hall voltage V_H, is developed across the slice.

(a) (i) State the faces between which the Hall voltage V_H is developed. [1]

(ii) Explain why a constant voltage V_H is developed between the faces you have named in **(i)**. [4]

(b) Two slices have similar dimensions. One slice is made of a metal and the other slice is made of a semiconductor material.

For the same values of magnetic flux density and current, state which slice, if either, will give rise to the larger Hall voltage. Explain your reasoning. [2]

[N17/P4/Q8]

Learning CORNER

Suggested Solution:

(a) (i) CFSP and DERQ.

(ii) Charge carrier experience a magnetic force normal to current and magnetic field. So negative charges will accumulate at side CFSP and positive charges will induce at DERQ. So an electric field is established between these parallel faces. Accumulation of charges stops when left side magnetic force is equal to right side electric force and a potential difference is developed across parallel faces.

(b) Since $V_H = \dfrac{BI}{ntq}$

Since B/T, I/A, t/m, q/C are constants for both slices,

So, $V_H \propto \dfrac{I}{n}$

Number density (n) of charge carrier is smaller in semiconductors, so V_H is larger for semiconductor slice.

Question 20

(a) State what is meant by a *field of force*. [2]

(b) Explain the use of a uniform magnetic field and a uniform electric field for the selection of the velocity of charged particles. You may draw a diagram if you wish. [4]

(c) A beam of charged particles enters a region of uniform magnetic and electric fields, as illustrated in Fig. 9.1.

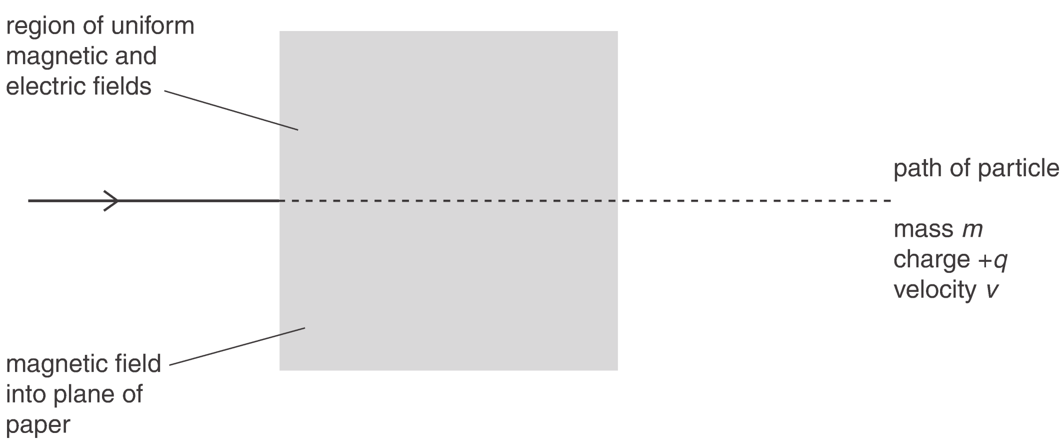


Fig. 9.1

The direction of the magnetic field is into the plane of the paper. The velocity of the charged particles is normal to the magnetic field as the particles enter the field.

A particle in the beam has mass m, charge $+q$ and velocity v. The particle passes undeviated through the region of the two fields.

On Fig. 9.1, sketch the path of a particle that has

(i) mass m, charge $+2q$ and velocity v (label this path Q), [1]

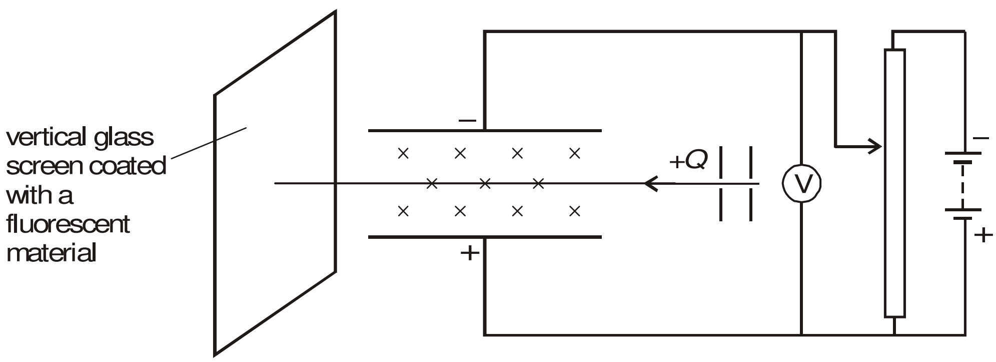
(ii) mass m, charge $+q$ and velocity slightly larger than v (label this path V). [2]

[N17/P4/Q9]

Suggested Solution:

(a) A 3-D region or space around an object where another object experience a force.

(b)



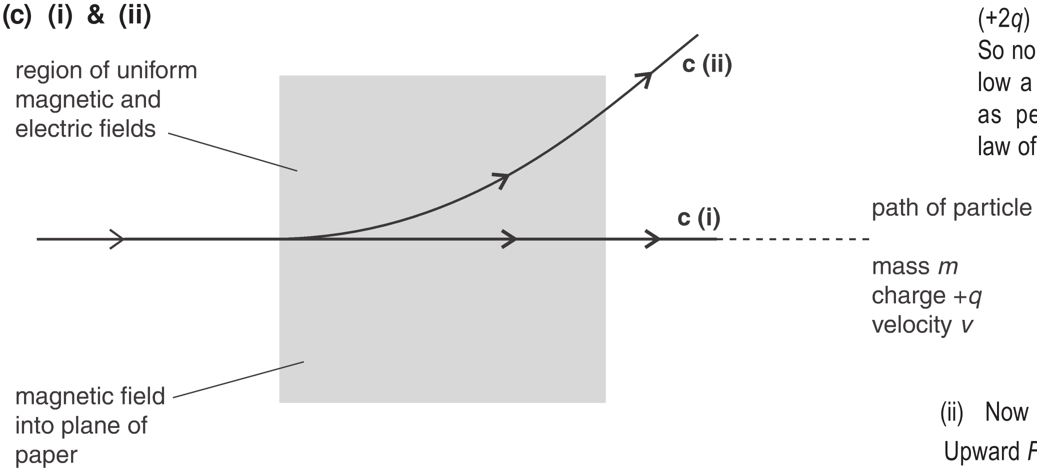
A beam of charged particles is projected normally in a mutually perpendicular electric and magnetic field. Adjust the potentiometer until a charged particle does not show any deflection and hit at the center of screen.

In equilibrium state, Upward F_E = Downward F_B

$$EQ = BQv\sin 90°$$

$$v = \frac{E}{B}$$

(c) (i) & (ii)



(c) (i)

Upward F_B = Downward F_E

$$BQV = EQ$$

Forces are independent of mass and charge ($+2q$) is cancelled out. So no deviation and follow a straight line path as per Newton's first law of motion.

(ii) Now

Upward F_B > Downward F_E

Hence a curved path in upward direction is traced.

Learning CORNER

Question 39

A heavy aluminium disc has a radius of 0.36 m. The disc rotates with the wheels of a vehicle and forms part of an electromagnetic braking system on the vehicle.

In order to activate the braking system, a uniform magnetic field of flux density 0.17 T is switched on. This magnetic field is perpendicular to the plane of rotation of the disc, as shown in Fig. 6.1.

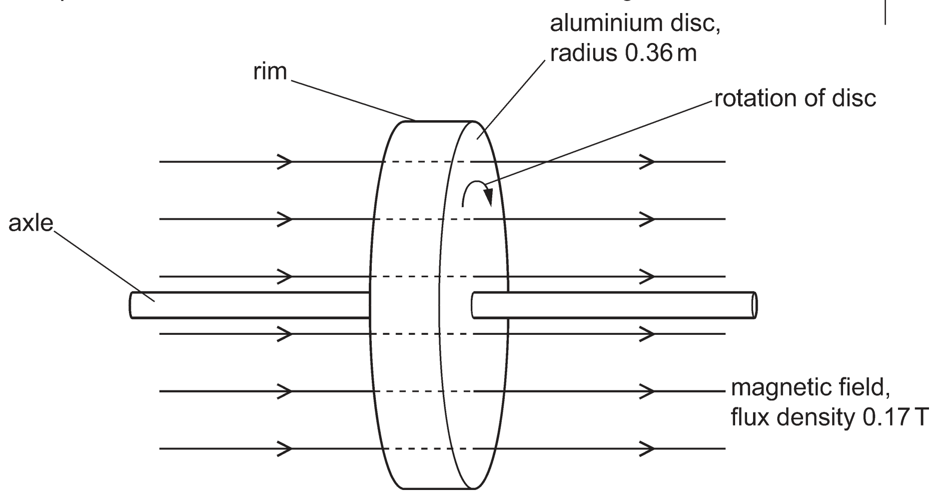


Fig. 6.1

(a) **(i)** Define magnetic flux. [2]

(ii) Calculate the magnetic flux through the disc. Give a unit with your answer. [2]

(b) The disc is rotating at a rate of 25 revolutions per second.

Calculate the magnitude of the electromotive force (e.m.f.) induced between the axle and the rim of the disc. [3]

(c) The axle and the rim are connected into an external circuit that enables the energy of the rotation of the disc to be stored for future use. The direction of rotation is shown in Fig. 6.1.

Use Lenz's law of electromagnetic induction to determine whether the current in the disc is from the rim to the axle or from the axle to the rim. Explain your reasoning. [3]

[J23/P4/Q6]

Suggested Solution:

(a) **(i)** Magnetic flux is the product of magnetic flux density and area perpendicular to magnetic field.

(a) (i) $\phi = (B)(A)$

(ii) $\phi = (B)(A)$

$= (B)(\pi r^2)$

$= (0.17)(3.14)(0.36)^2 = 0.0692$

$\therefore$ Magnetic flux $= 6.92 \times 10^{-2}$ Wb

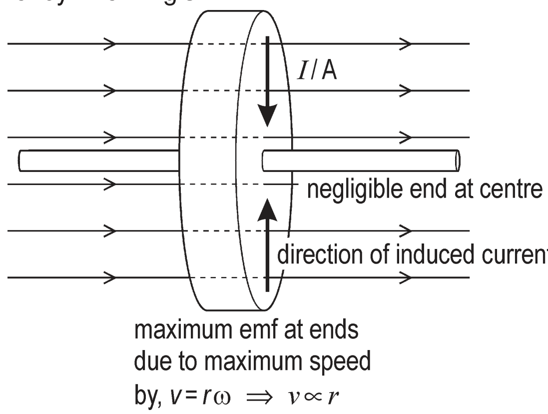
(b) $E =$ Rate of change of magnetic flux cutting

$$E = -(\Delta\phi)\left(\frac{\text{No. of revolutions}}{\Delta t}\right)$$

$$= -(6.92\times10^{-2})(25) = -1.73\text{ V}$$

$\therefore$ Magnitude of $E = 1.73$ V

(c) Induced current in the disc is always perpendicular to the magnetic field and causes a force to oppose the rotation of disc as per Lenz's law. So induced current is from rim to axle by Fleming's Left hand rule



Question 40

(a) A Hall probe containing a thin slice of semiconducting material is placed in a uniform magnetic field of flux density B. The largest faces of the slice are perpendicular to the magnetic field, as shown in Fig. 7.1.

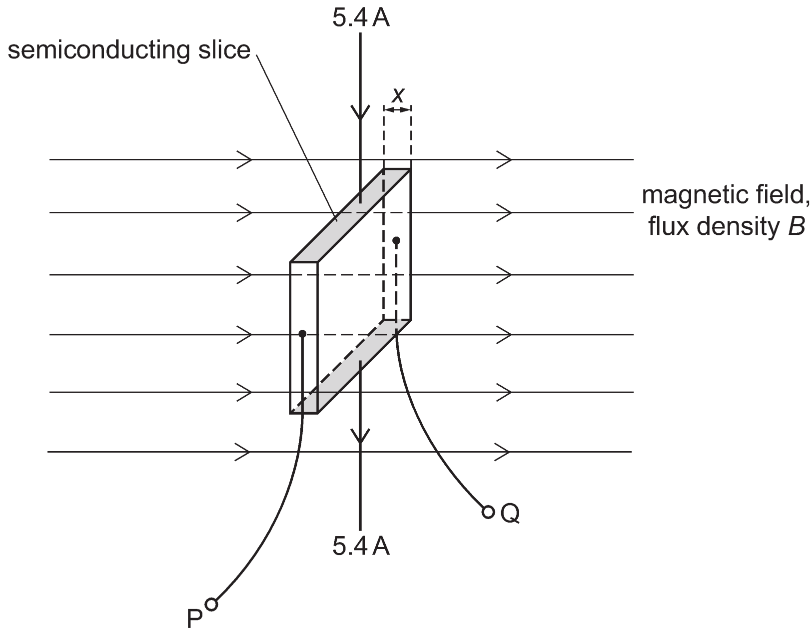


Fig. 7.1

The thickness x of the slice is 1.8 mm. The number density of charge carriers in the semiconducting material is $1.5 \times 10^{16}\text{ m}^{-3}$.

Learning CORNER

A constant current of 5.4 A is passed through the slice between the shaded faces.
The Hall voltage V_H that is developed between the terminals PQ is recorded.

Fig. 7.2 shows the variation with time t of B.

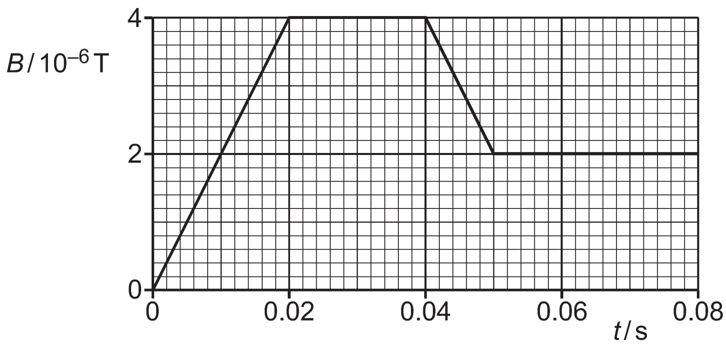


Fig. 7.2

(i) Show that, when B is equal to 4.0×10^{-6} T, the magnitude of V_H is 5.0 V. [1]

(ii) On Fig. 7.3, sketch the variation of V_H with t between $t = 0$ and $t = 0.080$ s.

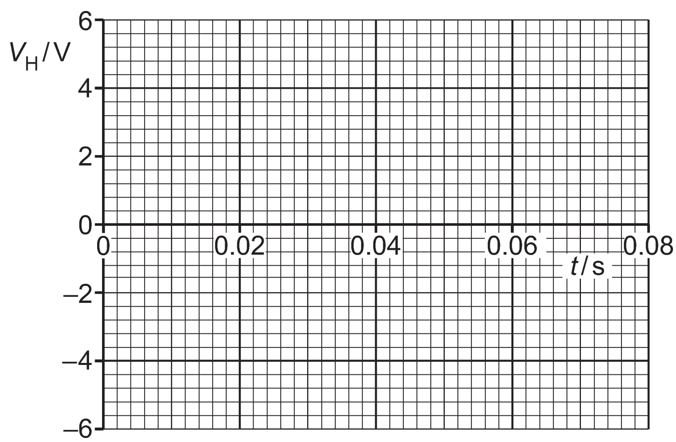


Fig. 7.3

[3]

(b) The Hall probe in **(a)** is replaced with a small flat coil that has 3000 turns. The cross-sectional area of the coil is 3.4×10^{-4} m^2.
The plane of the coil is perpendicular to the magnetic field. The electromotive force (e.m.f.) E induced between the terminals of the coil is recorded as B varies as shown in Fig. 7.2.

(i) Show that the magnitude of E at time $t = 0.010$ s is 2.0×10^{-4} V. [3]

(ii) On Fig. 7.4, sketch the variation of E with t between $t = 0$ and $t = 0.080$ s.

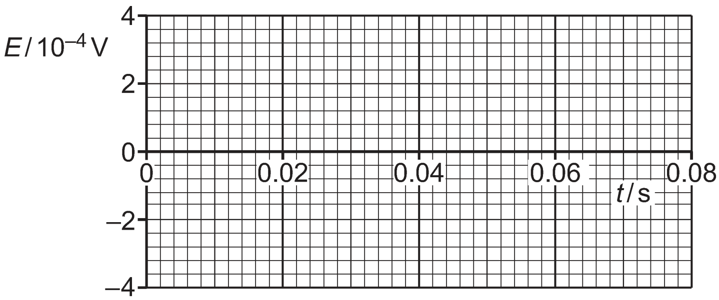


Fig. 7.4

[4]

[N23/P4/Q7]

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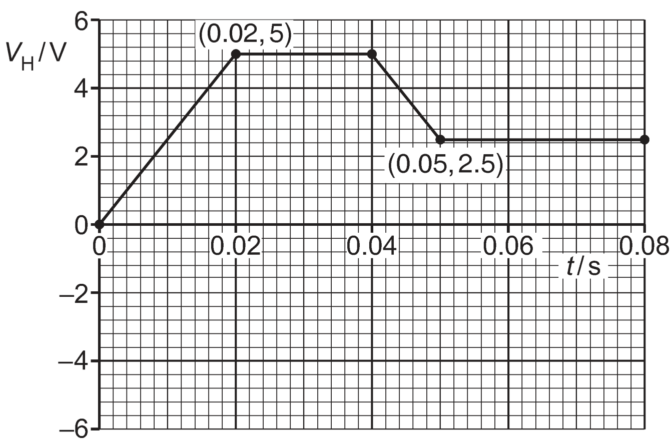
Suggested Solution:

(a) (i) $V_H = \dfrac{BI}{nqt}$

$$V_H = \frac{(4.0\times10^{-6})(5.4)}{(1.5\times10^{16})(1.60\times10^{-19})(1.8\times10^{-3})}$$

$V_H = 5.0$ V.

(ii)



(a) (ii) $V_H = \dfrac{BI}{nqt}$

$V_H = \left(\dfrac{I}{nqt}\right)B$

$V_H = (\text{constant})B$

$V_H \propto B$

So trend of graph should be same as shown in Fig. 7.2

$B / 10^{-6}$ T	0	4	2
V_H / V	0	5	2.5

(b) (i) emf = rate of change of magnetic flux linkage

$$\Rightarrow \quad E = -NA\left(\frac{\Delta B}{\Delta t}\right)$$

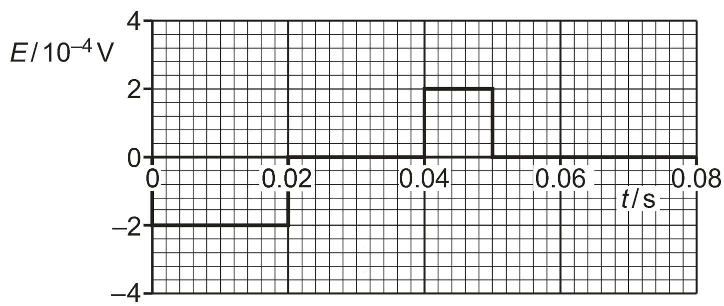
$$= -\frac{(3000)(3.4\times10^{-4})(2.0\times10^{-6})}{0.010}$$

$$= -2.04\times10^{-4} \text{ V}$$

Magnitude of induced e.m.f., $E = 2.04\times10^{-4}$ V

(b) (i) Value of B / T at $t = 0.010$ s is 2.0×10^{-6}.

(ii)



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(b) (ii)

$E = -NA\left(\frac{\Delta B}{\Delta t}\right)$

$E \propto -\left(\frac{\Delta B}{\Delta t}\right)$

$E \propto -$(Gradient of Fig. 7.2)

Question 41

(a) State Faraday's law of electromagnetic induction. [2]

(b) Fig. 7.1 shows a coil at rest in a uniform magnetic field that is parallel to the axis of the coil.

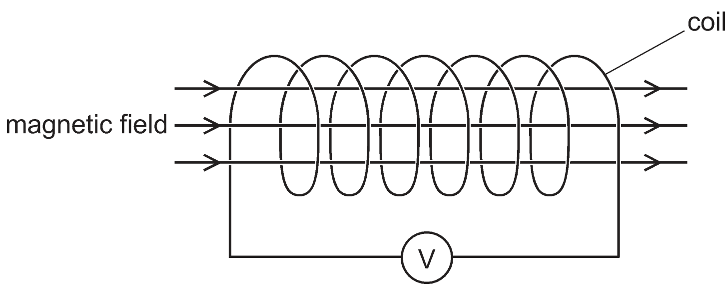


Fig. 7.1

The coil is connected to a centre-zero voltmeter.

The flux density B of the uniform magnetic field varies with time t as shown in Fig. 7.2.

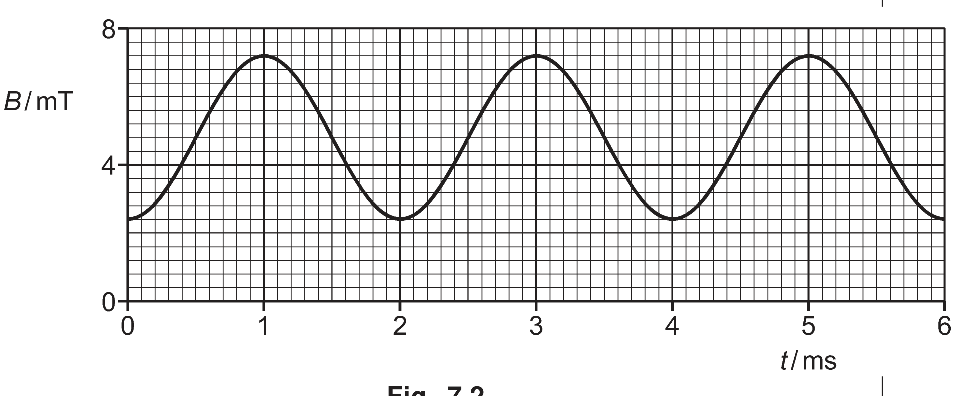


Fig. 7.2

The coil consists of 340 turns, each of cross-sectional area 3.2×10^{-4} m^2.

(i) Calculate the maximum magnetic flux through **one** turn of the coil. [2]

(ii) Determine the maximum rate of change of magnetic flux linkage in the coil. [3]

(iii) State the maximum electromotive force (e.m.f.) V_0 induced across the coil. [1]

(iv) On Fig. 7.3, sketch the variation of the e.m.f. V induced across the coil with t from $t = 0$ to $t = 6.0$ ms.

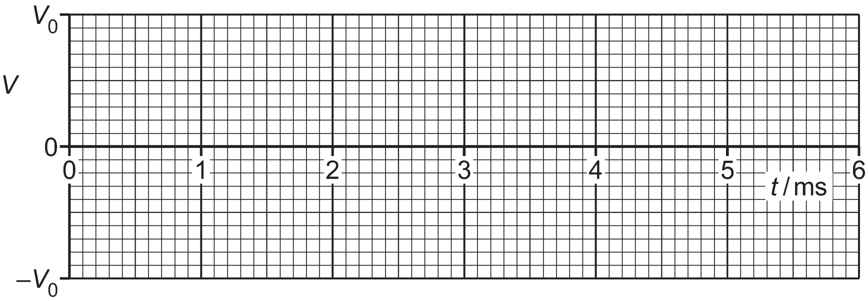


Fig. 7.3

[3]

(v) The variation of V with t can be described by

$$V = A \sin Bt$$

where A and B are constants.

Determine the values of A and B. Give units with your answers. [3]

[J24/P4/Q7]

Learning CORNER

Suggested Solution:

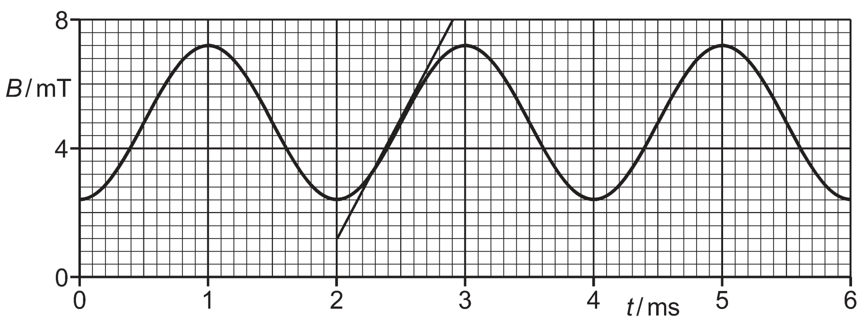
(a) Rate of change of magnetic flux linkage with a coil is directly proportional to the e.m.f. induced.

(a) $E \propto \dfrac{\Delta\phi}{\Delta t}$

(b) (i) Maximum magnetic flux, $\phi = NBA$

$$\phi = (1)(7.2\times10^{-3})(3.2\times10^{-4})$$
$$= 2.30\times10^{-6}\ \text{Wb}$$

(ii)



(b) (ii) Gradient is maximum if tangent is at $t = 0.5$ ms, 1.5 ms, 2.5 ms, 3.5 ms, 4.5 ms, 5.5 ms

$$\Delta\phi = (\text{No. of turns})\times\frac{(\text{change of flux linkage})}{\text{time}}\times(\text{cross-sectional area})$$

$$\Delta\phi = (\text{No. of turns})\times(\text{max. gradient of tangent line on graph})\times(\text{cross-sectional area})$$

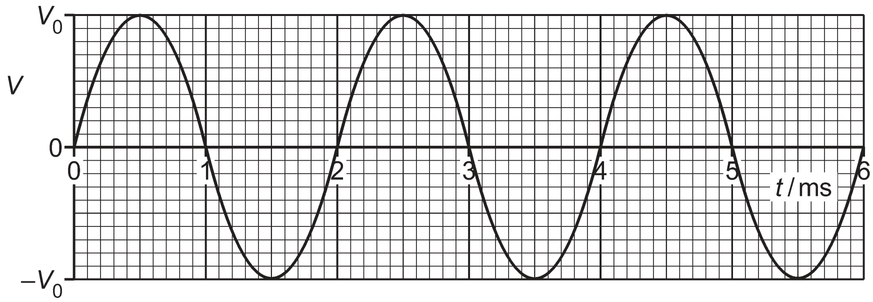
$$\Delta\phi = (340)\left(\frac{(8.0-1.2)\times10^{-3}}{(2.9-2.0)\times10^{-3}}\right)(3.2\times10^{-4})$$

$$\Delta\phi = 0.822$$

$\therefore$ Max. rate of change of flux linkage = 0.822 Wb s^{-1}

(iii) $V_0 = 0.822$ V

(iv)



(v) $V = V_0 \sin \omega t$

For A: $A = V_0 = 0.822$ V

For B: $B = \omega = \dfrac{2\pi}{T}$

$$= \frac{2(3.14)}{2.0 \times 10^{-3}} = 3.14 \times 10^{3} \text{ rad s}^{-1}$$

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(b) (iii) $\text{e.m.f} = V_0$

$$= -N\left(\frac{\Delta B}{\Delta t}\right)A$$

–ve sign is due to Lenz's Law.

But –ve sign can be neglected as per CAIE 9702 syllabus.

Question 42

A metal wheel consists of an axle A, eight spokes and a rim, as shown in Fig. 1.1.

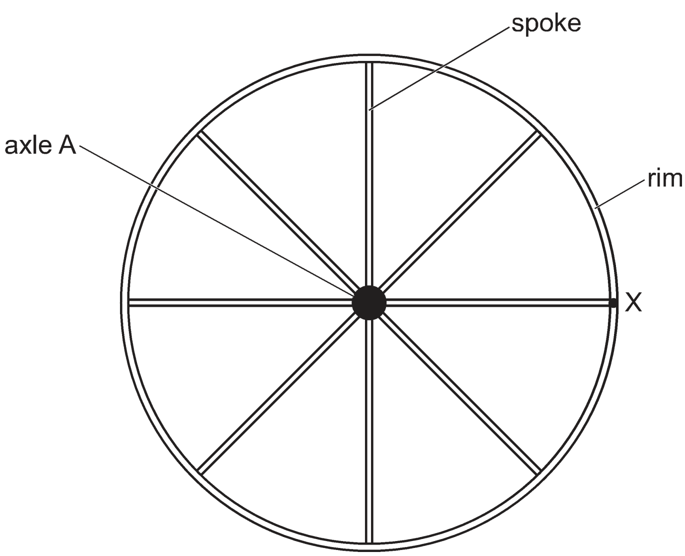


Fig. 1.1

Point X is on the rim at the end of one of the spokes.

The rim has a radius of 0.85 m.

The wheel is rotating clockwise with an angular speed of 140 rad s^{-1}.

(a) For point X, determine:

(i) the speed [2]

(ii) the centripetal acceleration. [2]

(b) There is a uniform magnetic field of flux density 0.18 T into the plane of the page.

(i) State Lenz's law of electromagnetic induction. [2]

(ii) Show that the time taken for point X to complete one revolution is 45 ms. [1]

(iii) Calculate the magnetic flux cut by spoke AX during one revolution of the wheel.

Give a unit with your answer. [3]

(iv) Determine the magnitude of the electromotive force (e.m.f.) induced across spoke AX. [2]

(v) Use Lenz's law to explain whether the potential is higher at end A or end X of the spoke. [1]

[N24/P4/Q1]

Learning CORNER

Suggested Solution:

(a) (i) speed, $v = r\omega$

$$= (0.85)(140)$$
$$= 119 \text{ m s}^{-1}$$

(ii) acceleration, $a = v\omega$

$$= (119)(140) = 16\ 660$$
$$\approx 1.67 \times 10^4 \text{ m s}^{-2}$$

(a) (ii) *Alternatively*

- $a = r\omega^2$
  $= (0.85)(140)^2$
  $= 16\ 660 \text{ ms}^{-2}$
- $a = \dfrac{v^2}{r}$
  $= \dfrac{(119)^2}{0.85}$
  $= 16\ 660 \text{ ms}^{-2}$

(b) (i) The magnetic effect of induced current always opposes the change producing it.

(ii) $\omega = \dfrac{2\pi}{T}$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} \quad \Rightarrow \quad T = \frac{2(3.14)}{140} = 0.044865 \text{ s} \approx 45 \text{ ms}$$

(iii) magnetic flux, $\phi = (B)(A)$

$$= (B)(\pi r^2)$$
$$= (0.18)(3.14)(0.85)^2$$
$$= 0.408 \text{ Wb}$$

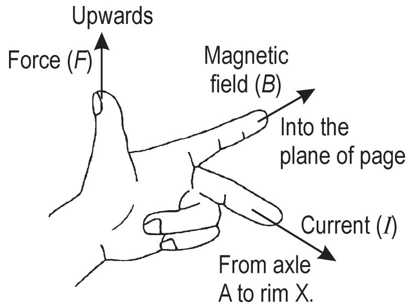
(iv) Induced e.m.f., $E = \dfrac{\Delta\phi}{\Delta t}$

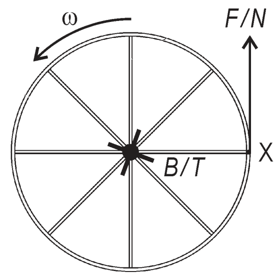
$$E = \frac{0.408}{45 \times 10^{-3}} = 9.07 \text{ V}$$

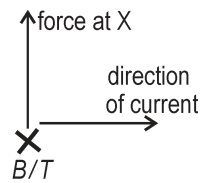
(v) Since, wheel is rotating clockwise, so force at spoke is anti-clockwise or at X is upward by Lenz's law. By Fleming's left hand rule, current is directed from axle A to end X of the spoke. Hence, X is at the higher potential.

Teacher's Comments

Learning CORNER







Centre axle A hardly moves and spoke end X move at high speed. Hence different parts of spoke move with different velocities.